Aerodynamic Shape Optimization with Goal-Oriented Error Estimation and Control

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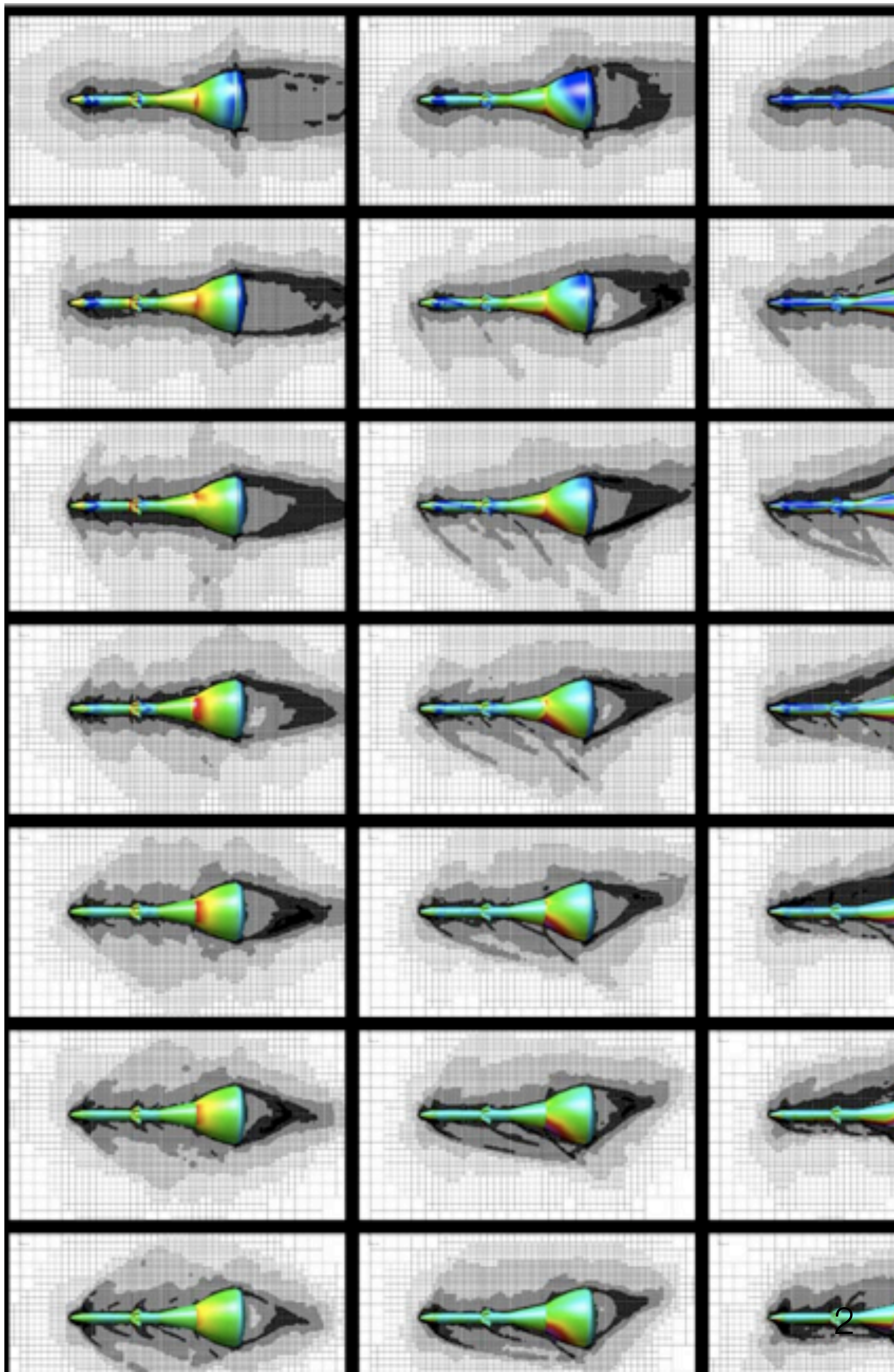
Motivation

- Challenges of simulation-based design
  - High CFD expertise in mesh generation
    - Long setup time
    - High cost due to repeated flow solves on fine meshes or high uncertainty due to inappropriate meshes
- Success of error estimation and mesh adaptation in goal-oriented simulations



Objectives

Adaptive discretization of aerodynamic shape optimization problems

Accuracy

- Improve design confidence
  - Direct control over objective function discretization error

Automation

- Reduce level of CFD expertise
  - Eliminate the need to handcraft a mesh appropriate for all candidate designs
  - Shorten problem setup time

Progress toward improved efficiency

- Reduce cost by systematically increasing depth of refinement as designs improve
  - Progressive optimization strategy

Problem Formulation

$$\min_{X} \quad J(X, \mathbf{Q})$$

subject to

$$R(X, \mathbf{Q}) = 0 \qquad \forall X \in \Omega$$

- Gradient-based optimization

$$\frac{\mathrm{d}J}{\mathrm{d}X} \longrightarrow 0$$

- Steady Euler equations

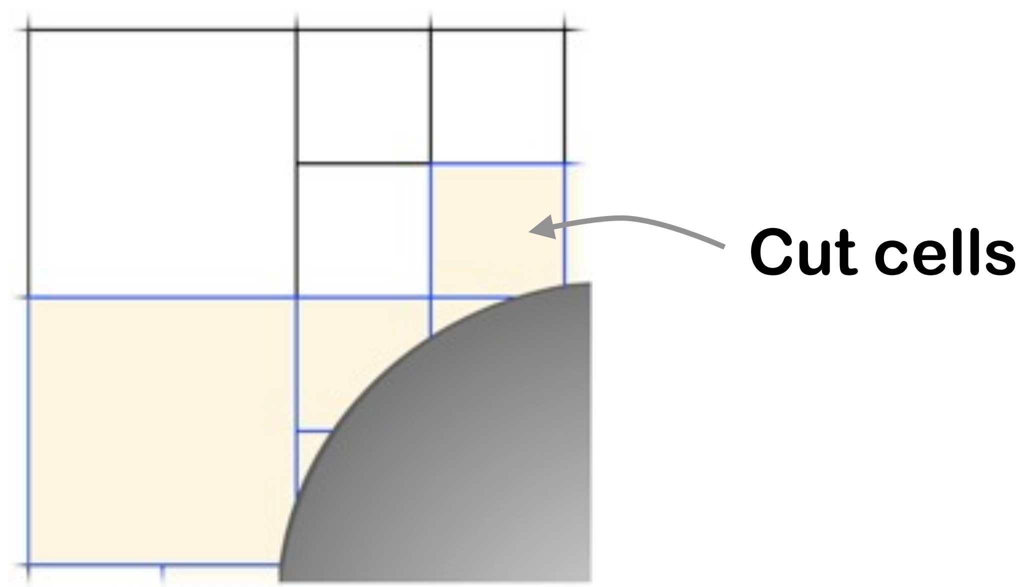
Spatial Discretization: $J_H(X, \mathbf{Q}_H)$, $R_H(X, \mathbf{Q}_H)$

- Second-order finite-volume method
- Cartesian mesh with embedded boundaries

✓Complex geometry

✓Automation

✓*h*-refinement



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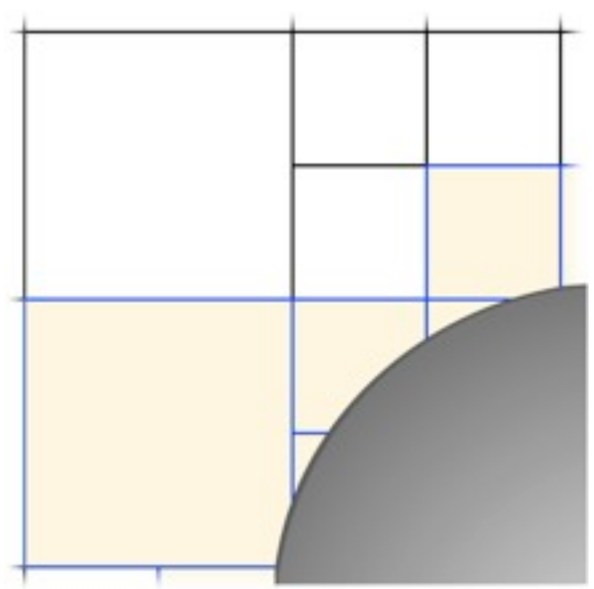
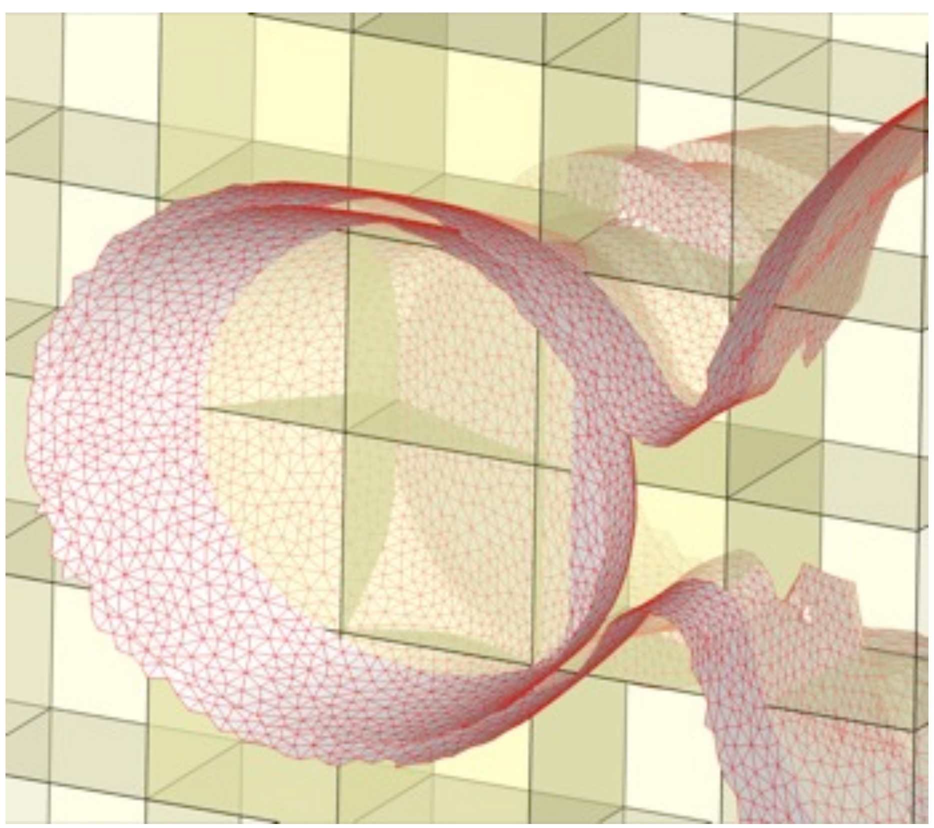
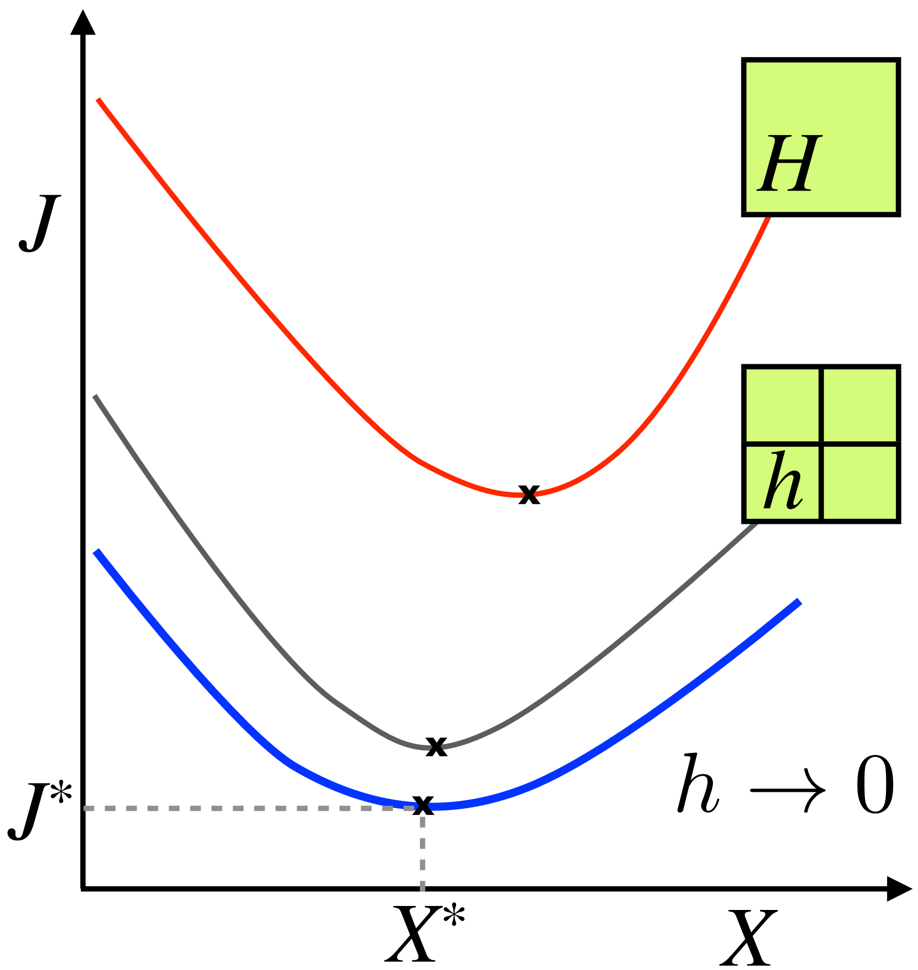


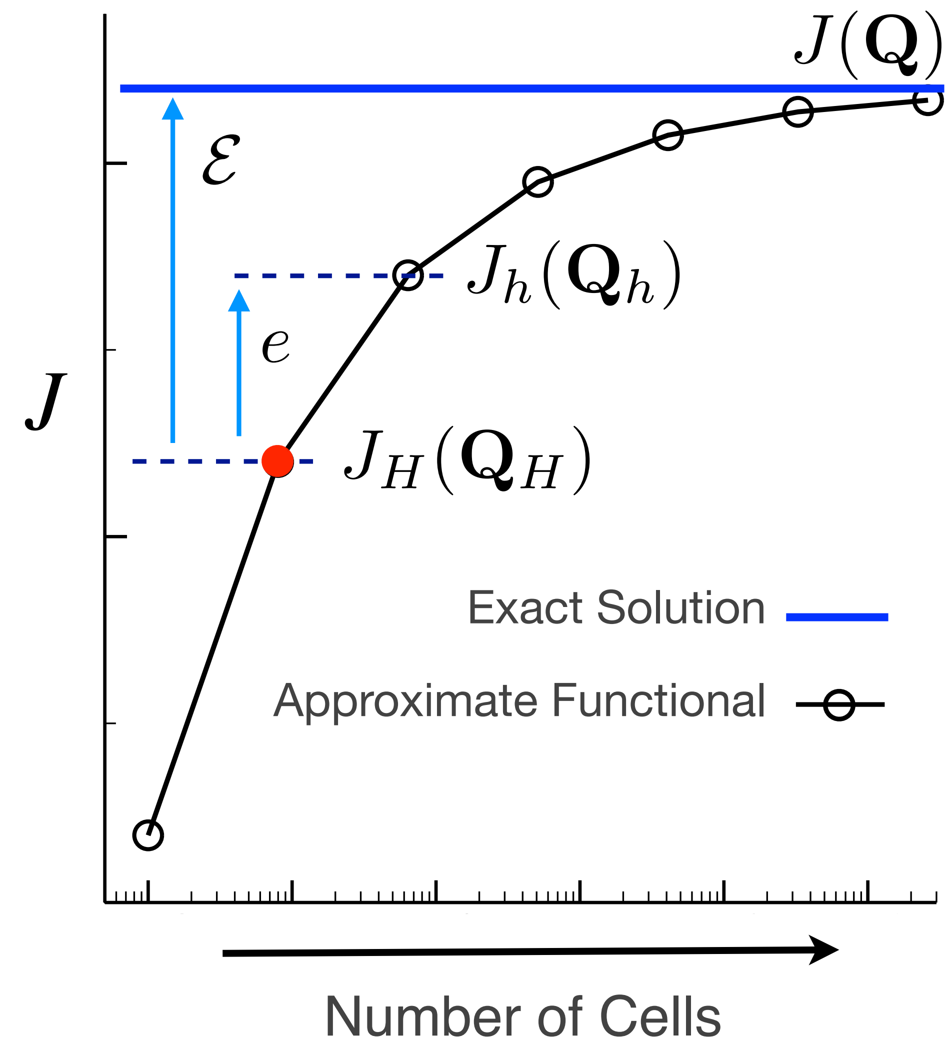


Discretization Error

Design Space



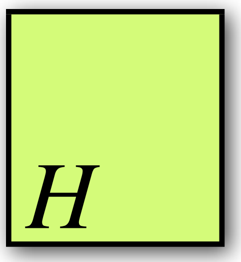
Error Estimate (fixed X)



- Leverage adjoint method
  - Error estimates via the method of adjoint weighted residuals
  - Objective function gradient via the discrete adjoint method

Dual Role of Adjoints

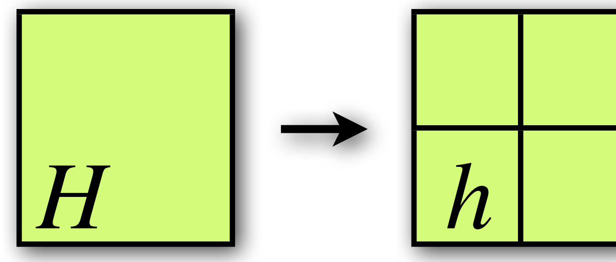
Gradients



$$J_H = f(X, \mathbf{Q}_H)$$

$$\text{e.g. } C_D + (C_L - C_L^*)^2$$

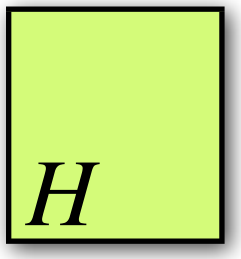
Error Estimates



$$e = |J_h - J_H)|$$

Dual Role of Adjoints

Gradients



$$J_H = f(X, \mathbf{Q}_H)$$

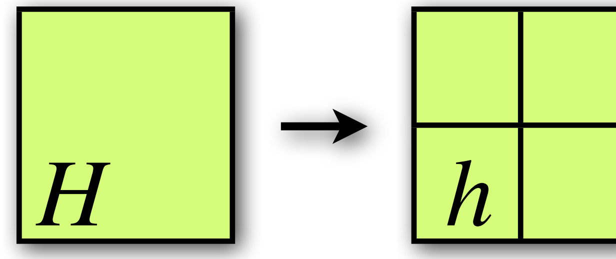
e.g. $C_D + (C_L - C_L^*)^2$

$$\frac{\mathrm{d}J}{\mathrm{d}X} = \frac{\partial J}{\partial X} + \frac{\partial J}{\partial \mathbf{Q}}\frac{\mathrm{d}\mathbf{Q}}{\mathrm{d}X}$$

$$0 = \frac{\partial \mathbf{R}}{\partial X} + \frac{\partial \mathbf{R}}{\partial \mathbf{Q}}\frac{\mathrm{d}\mathbf{Q}}{\mathrm{d}X}$$

$$\frac{\mathrm{d}J}{\mathrm{d}X} = \frac{\partial J}{\partial X} - \psi^{\mathrm{T}}\frac{\partial \mathbf{R}}{\partial X}$$

Error Estimates



$$e = |J_h - J_H)|$$

$$J_h \approx J_h(\mathbf{Q}_H) + \frac{\partial J(\mathbf{Q}_H)}{\partial \mathbf{Q}}\Delta\mathbf{Q}$$

$$0 \approx R_h(\mathbf{Q}_H) + \frac{\partial R(\mathbf{Q}_H)}{\partial \mathbf{Q}}\Delta\mathbf{Q}$$

$$J_h \approx J_h(\mathbf{Q}_H) - \psi^{\mathrm{T}}\mathbf{R}_h(\mathbf{Q}_H)$$

$$\boxed{\left[\frac{\partial \mathbf{R}}{\partial \mathbf{Q}}\right]^{\mathrm{T}}\psi = \frac{\partial J}{\partial \mathbf{Q}}}$$

Error Estimation Details

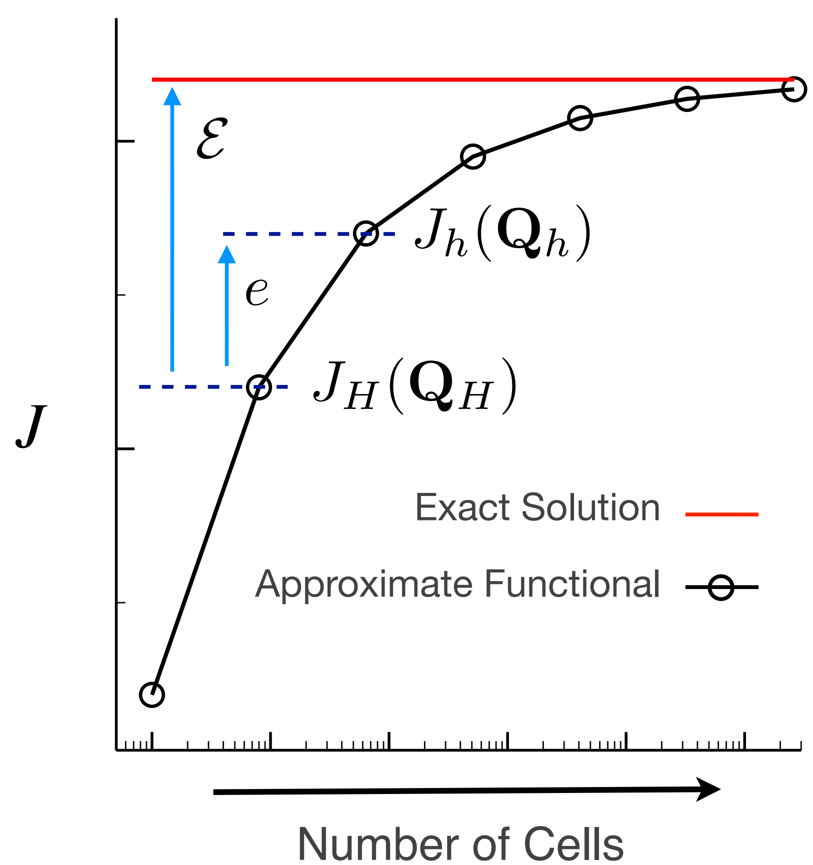
$$J_h(\mathbf{Q}_h) \approx J_h(\mathbf{Q}_H) - \psi_h^T \, \mathbf{R}_h(\mathbf{Q}_H)$$

$$J_c = J_h(\mathbf{Q}_H) - \psi_H^T \, \mathbf{R}_h(\mathbf{Q}_H)$$

$$\boldsymbol{\eta}_H = \left(\tilde{\psi}_h - \psi_H\right)^T \mathbf{R}_h(\mathbf{Q}_H)$$

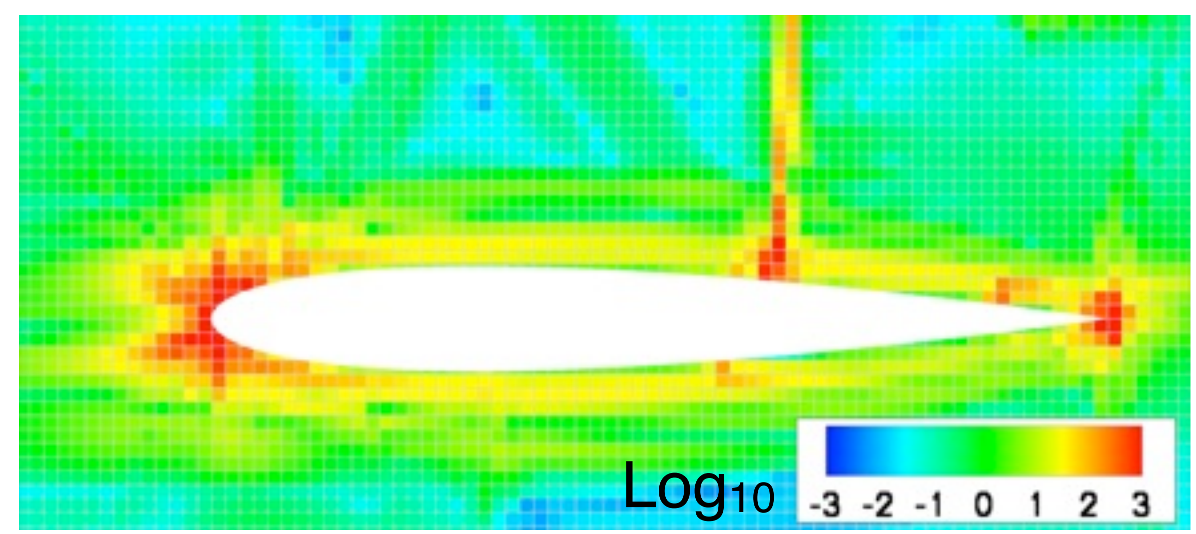
Error Estimate

$$\mathcal{E} = C \, |J_c - J_H|$$

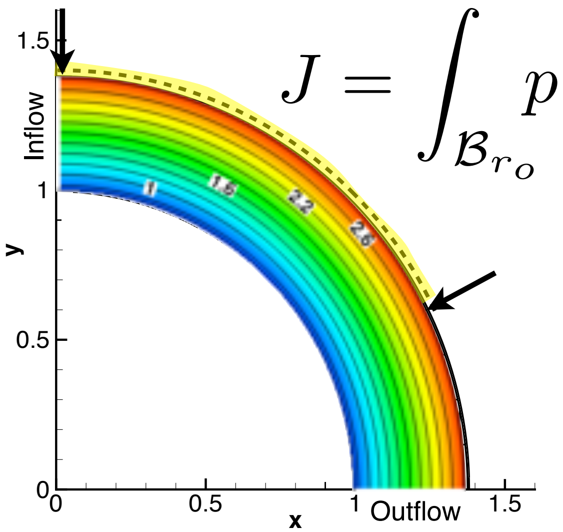


Refinement Indicator

$$\eta = \sum_{i=1}^{N} |\boldsymbol{\eta}_i|$$

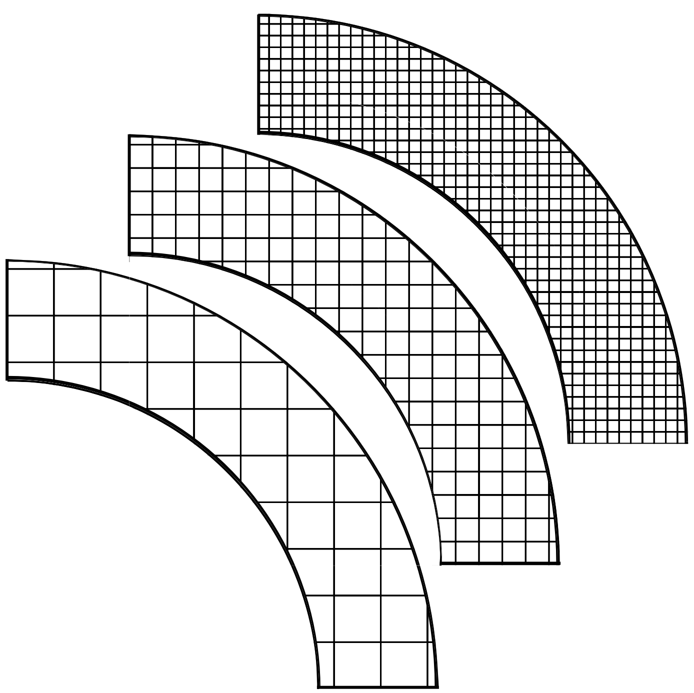


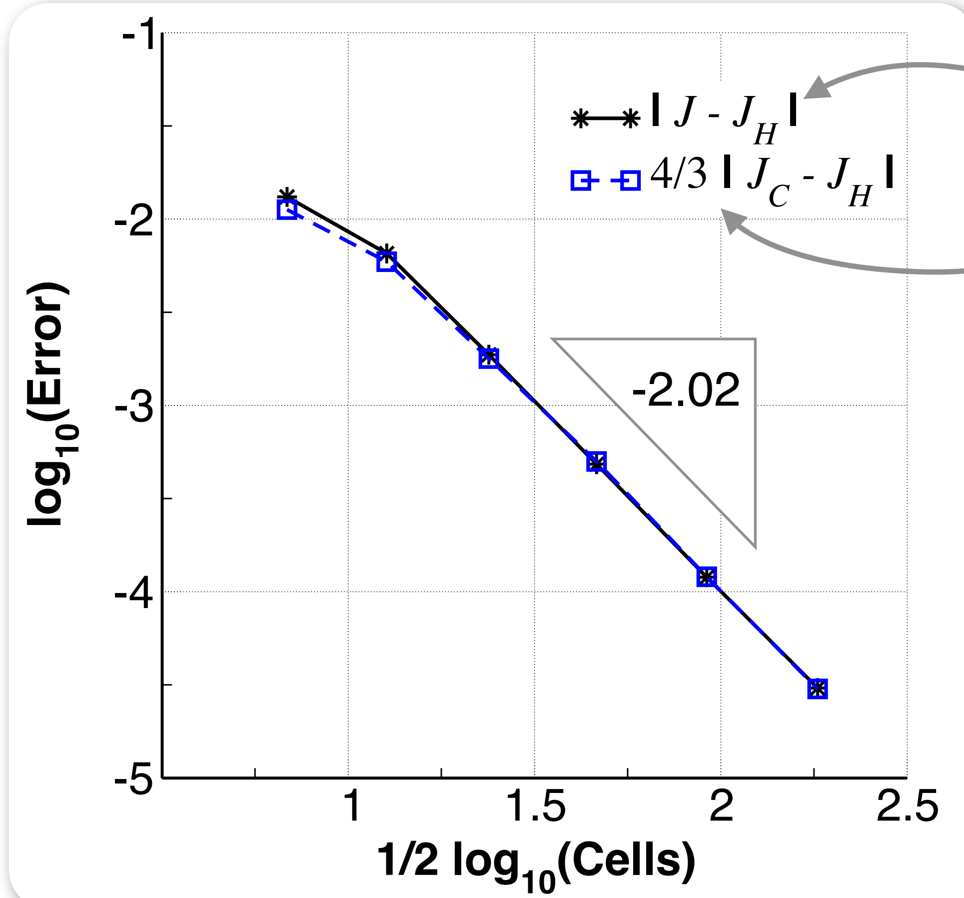
Verification: Supersonic Vortex



$$J = \int_{\mathcal{B}_{r_o}} p \ \mathrm{d}l$$

Uniform Refinement





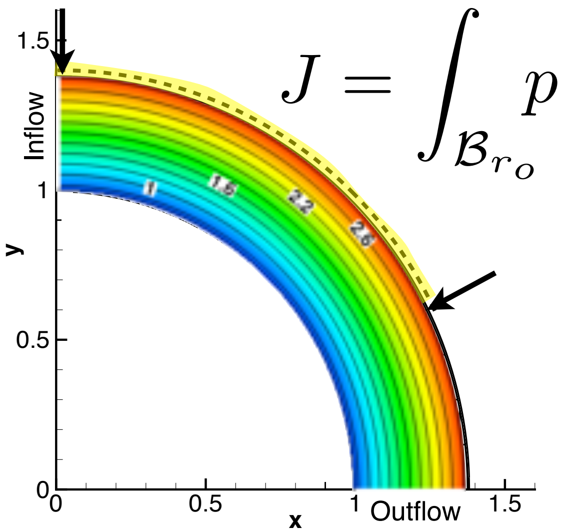
True Error

Error Estimate

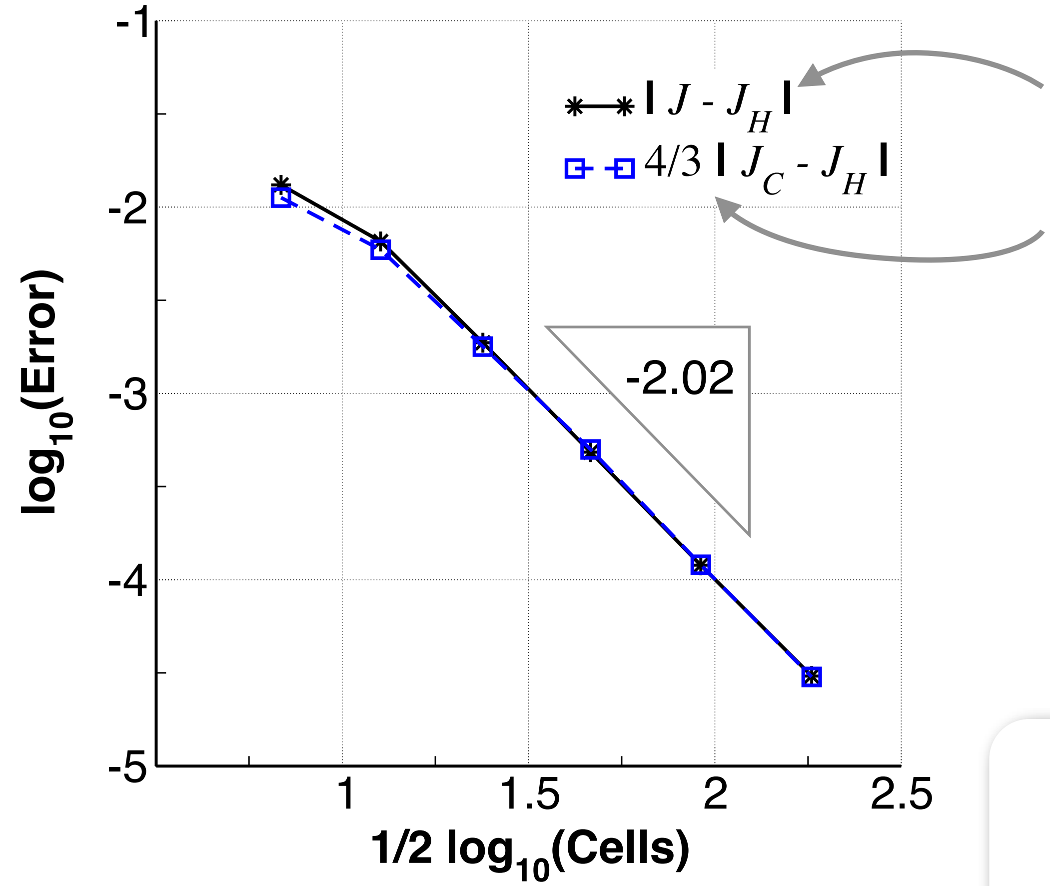
$$\mathcal{E} = C\ |J_c - J_H|$$

- No limiter, $\mathcal{O}(h^2)$
- Effectivity close to 1

Verification: Supersonic Vortex



$$J = \int_{\mathcal{B}_{r_o}} p \, \mathrm{d}l$$



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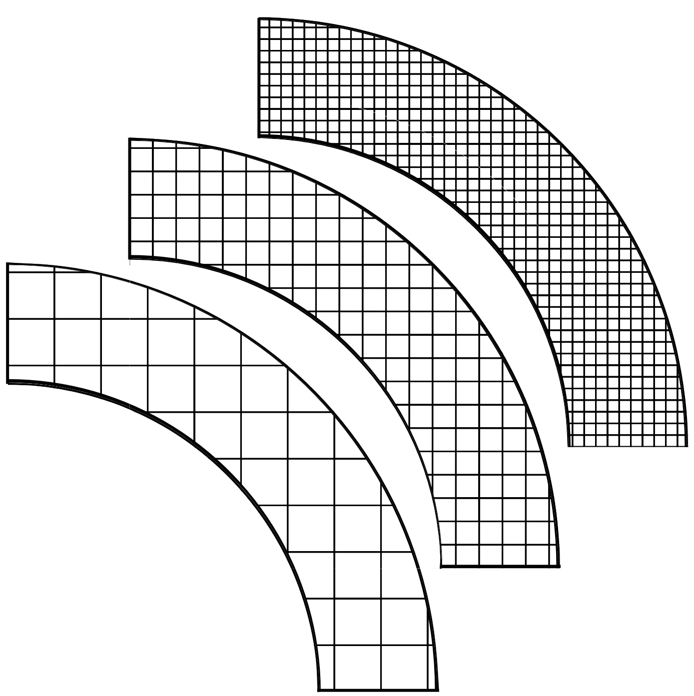
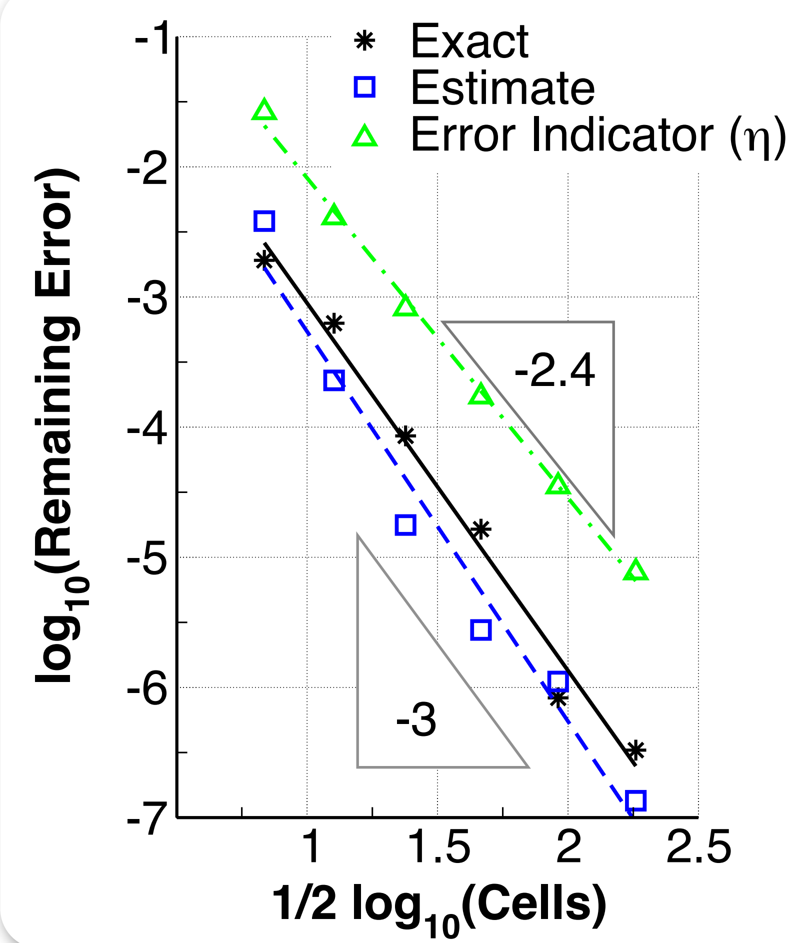


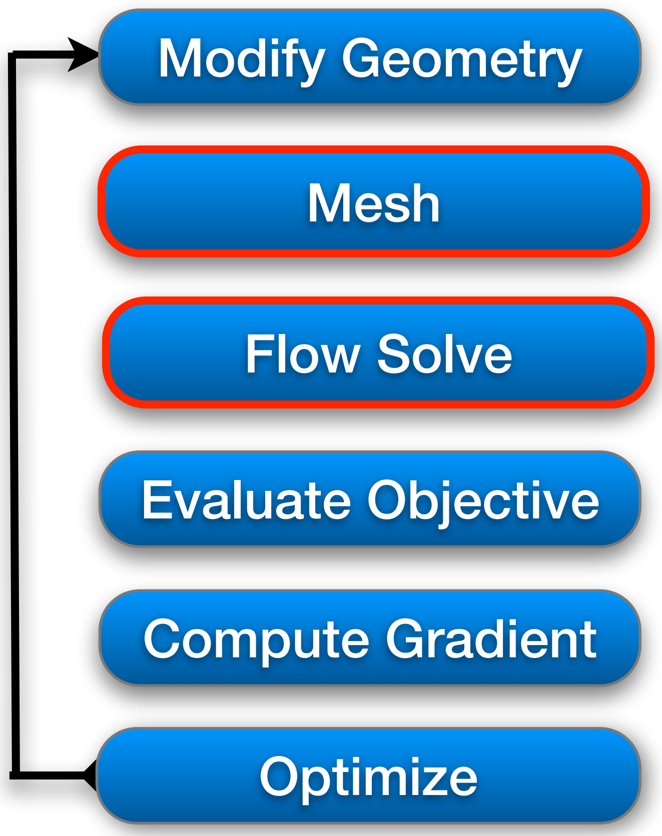
Refinement Indicator

$$\eta_H = \left| \left(\tilde{\psi}_h - \psi_H \right)^T \mathbf{R}_h(\mathbf{Q}_H) \right|$$

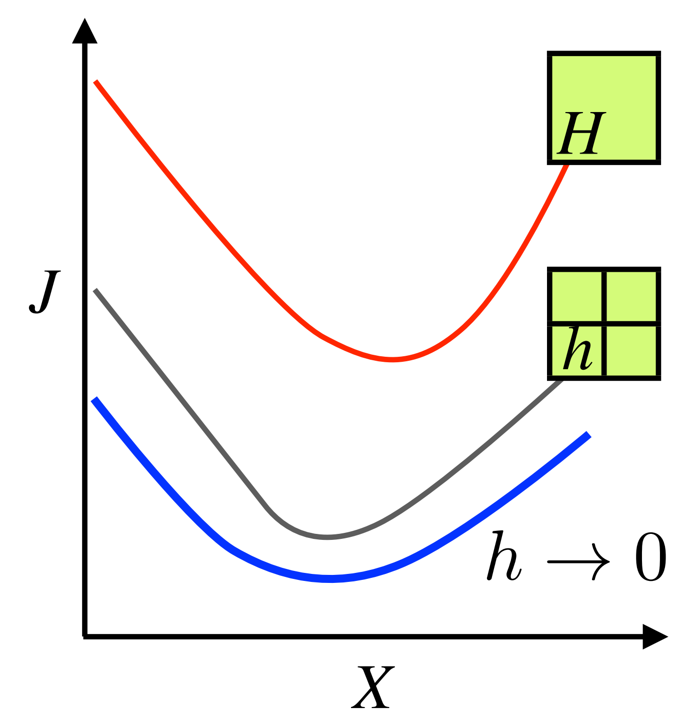
- Sharp estimate of remaining error
- Localization very conservative



Optimization with Mesh Adaptation



- Integration into existing, fixed mesh, optimization framework
  - Build sequence of adapted meshes
  - Pass values of objective and gradient from finest mesh to optimizer



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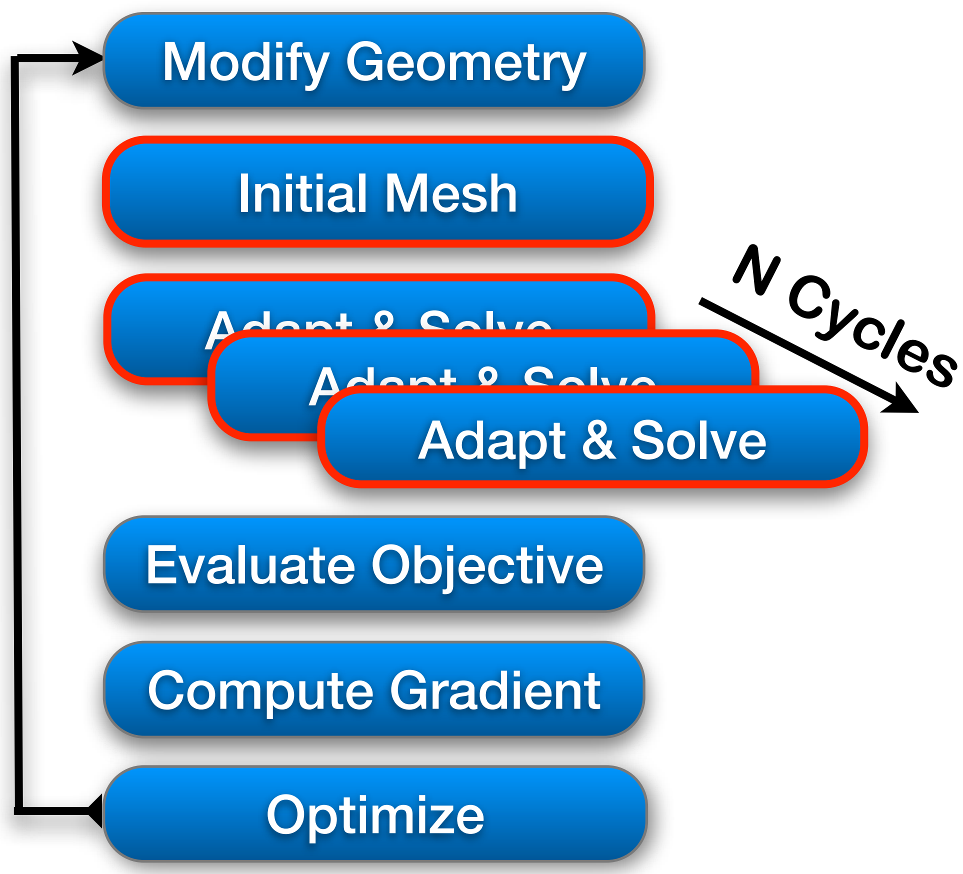

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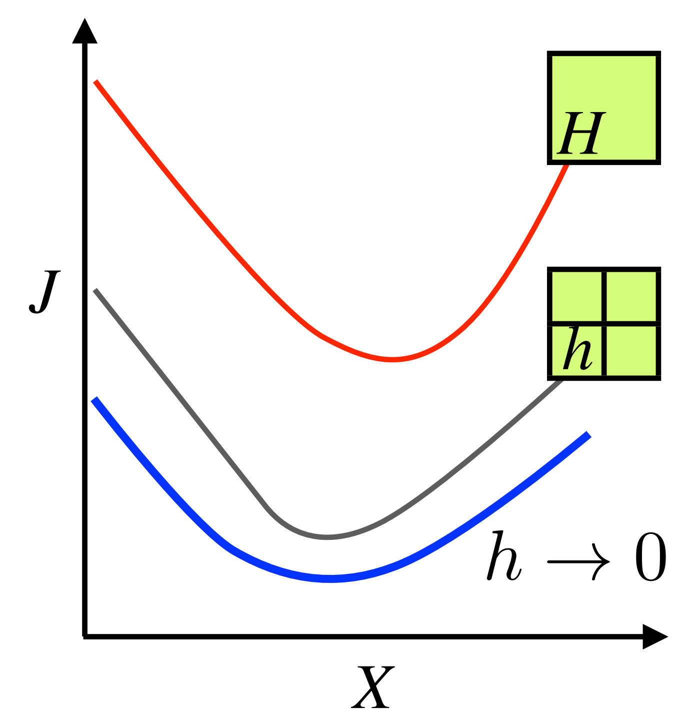
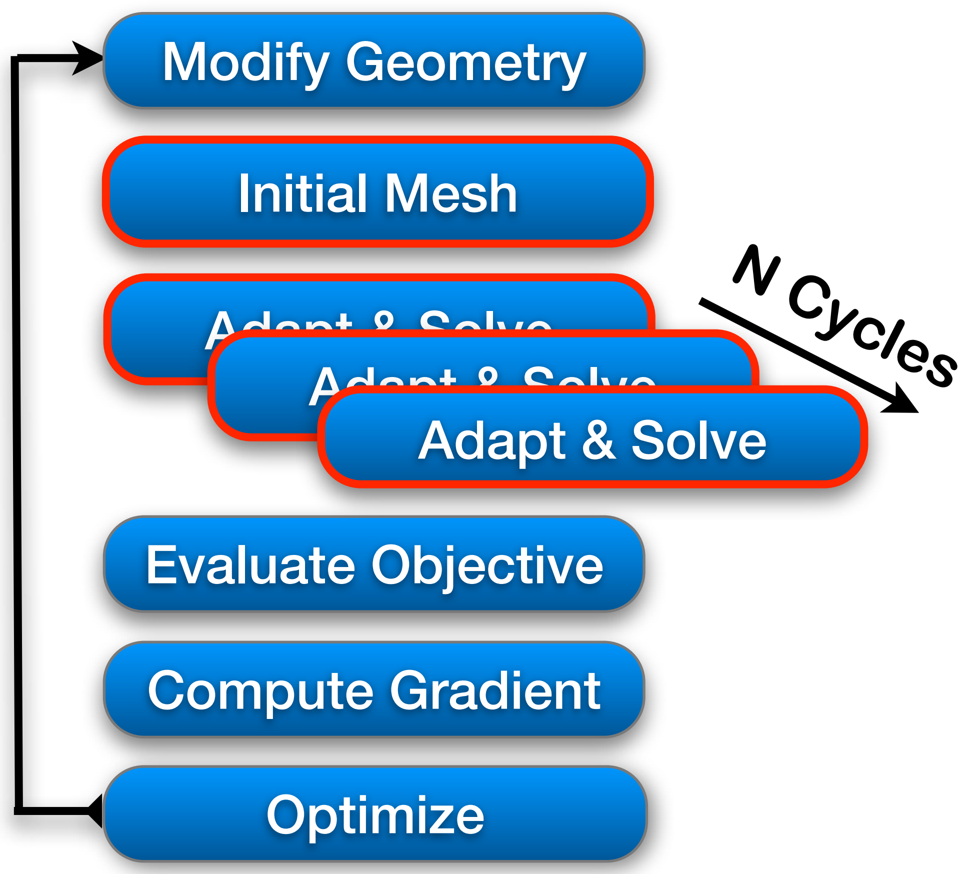
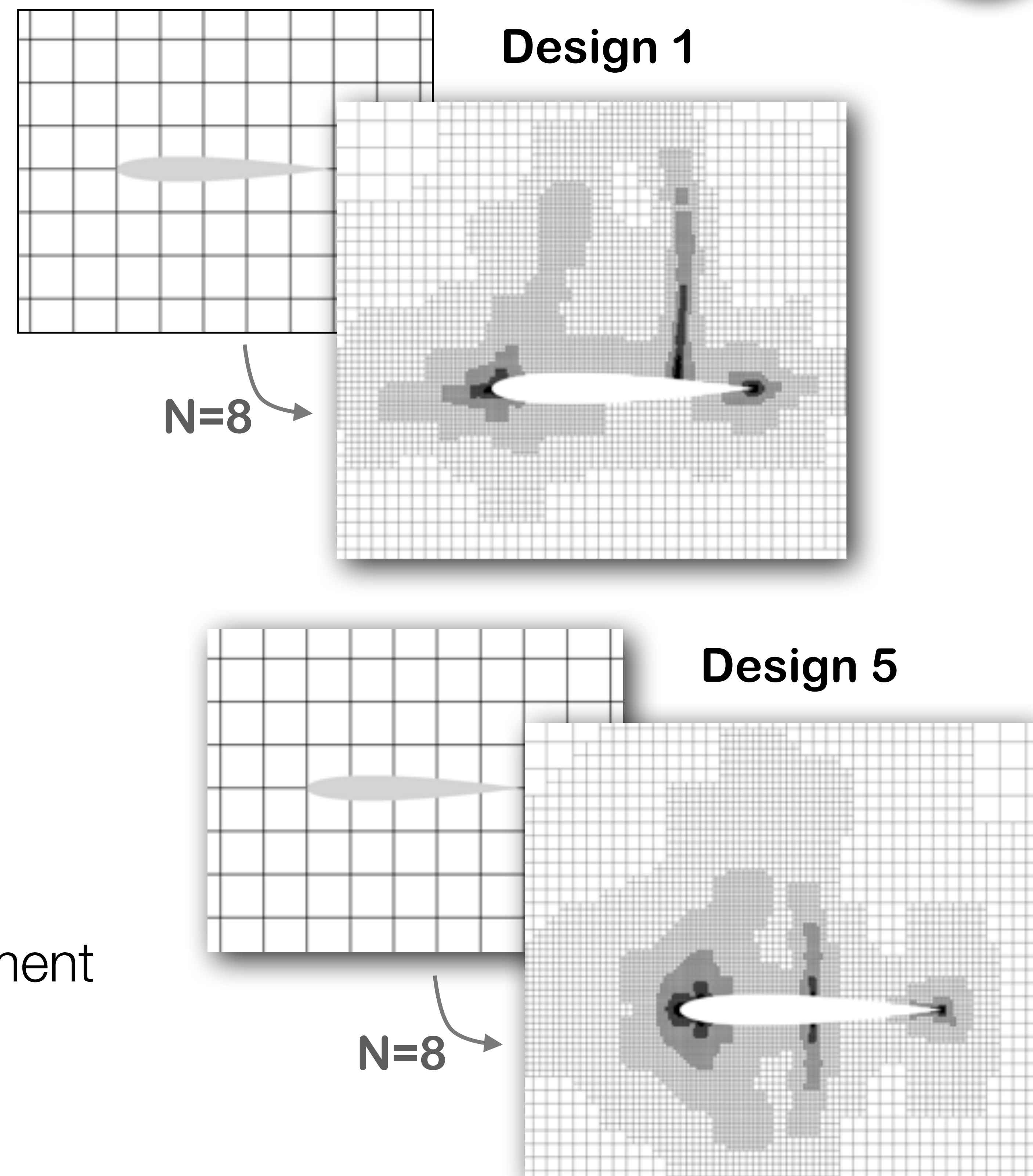


- In each design iteration, perform fixed (user specified) number of adaptations
  - Fixed depth strategy
  - Robust and precise control over computational resources

Optimization with Mesh Adaptation

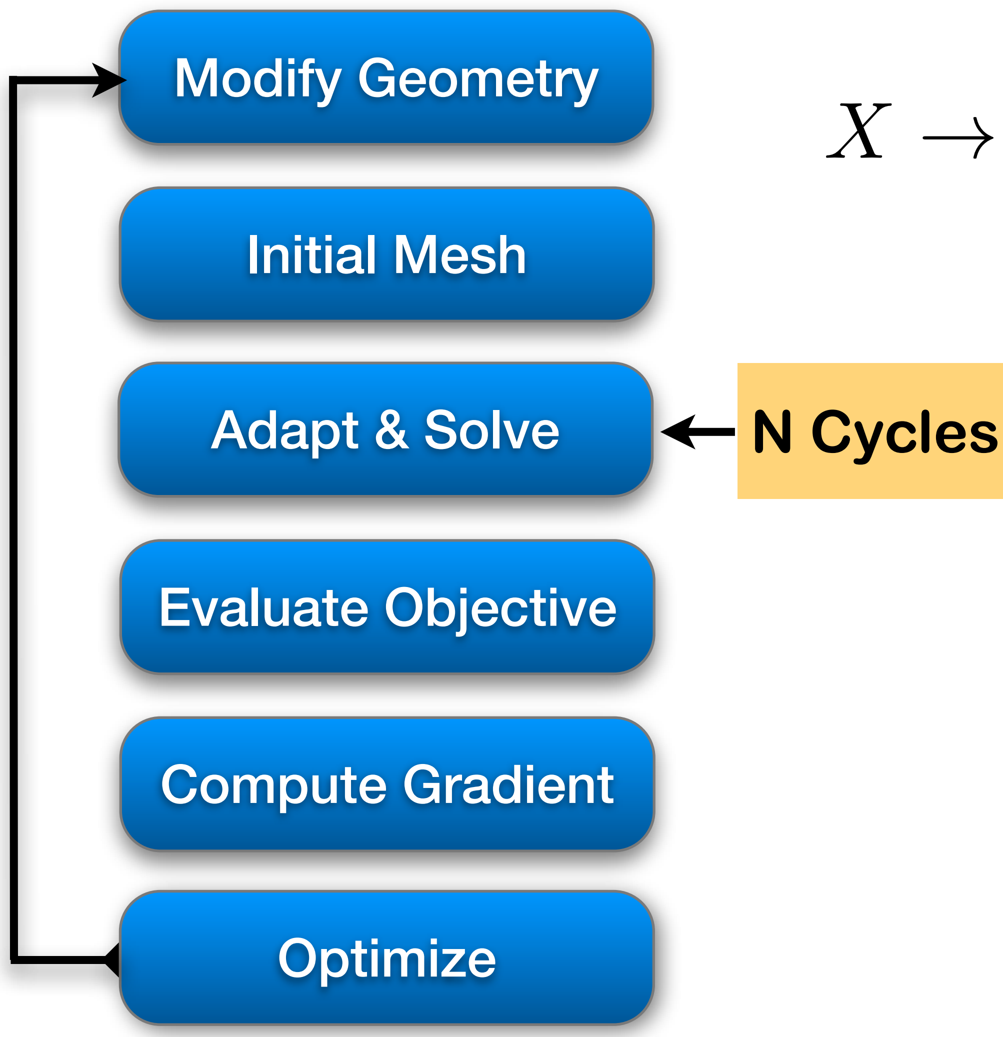




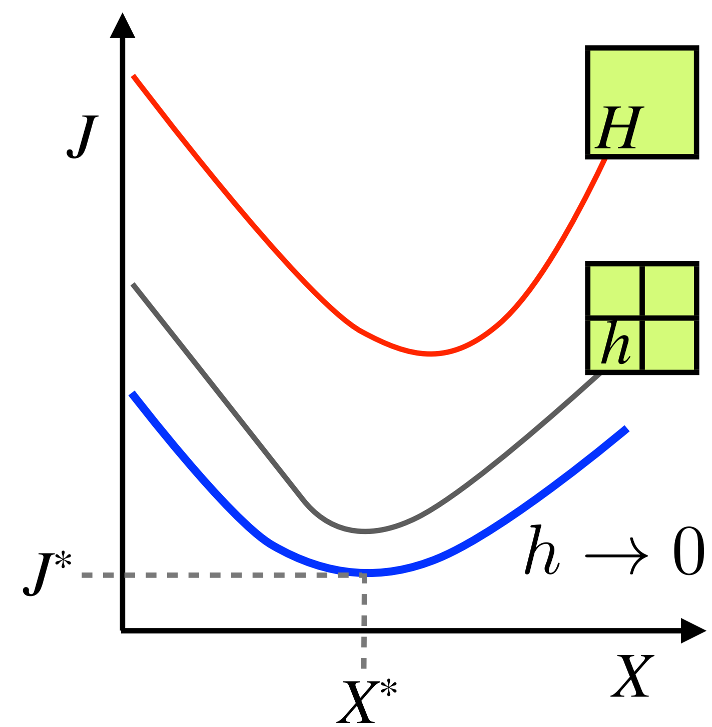
- In each design iteration:
  - Start with same initial mesh
  - Adapt until prescribed refinement level is attained
- May be inefficient

Progressive Optimization

- Increase mesh refinement in each optimization subproblem
  - Converge a sequence of improving discretizations



$$X \to X^* \text{ as } \mathcal{E} \to 0$$



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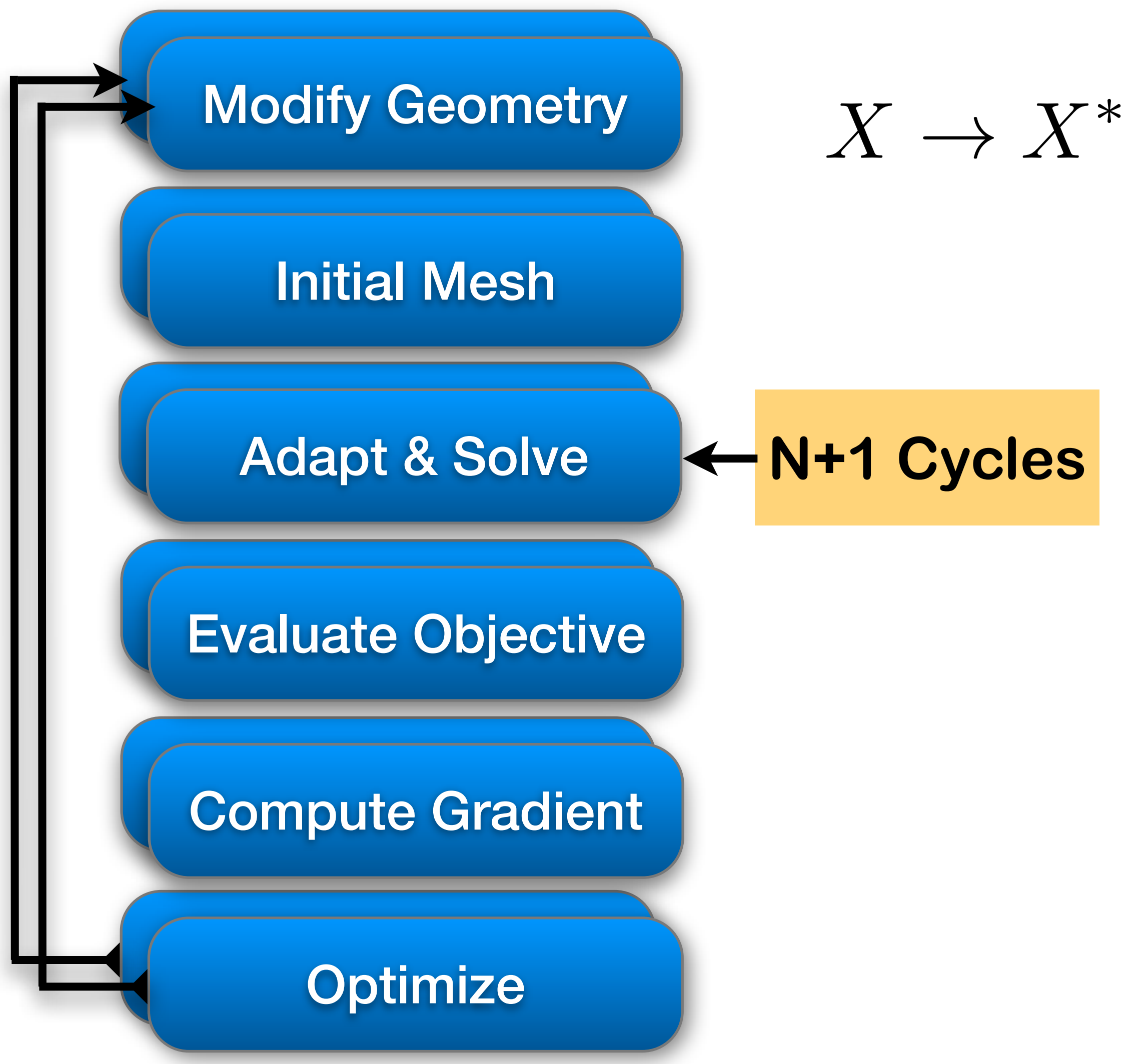

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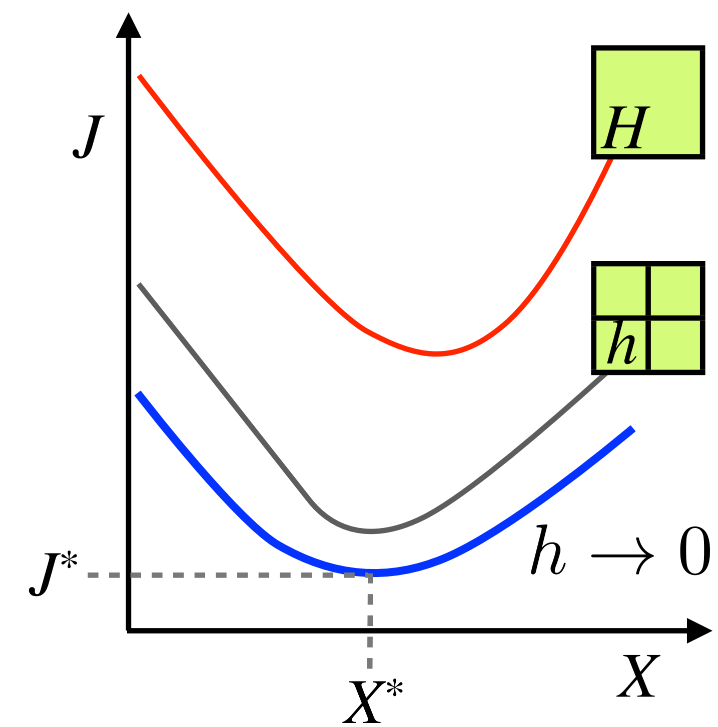

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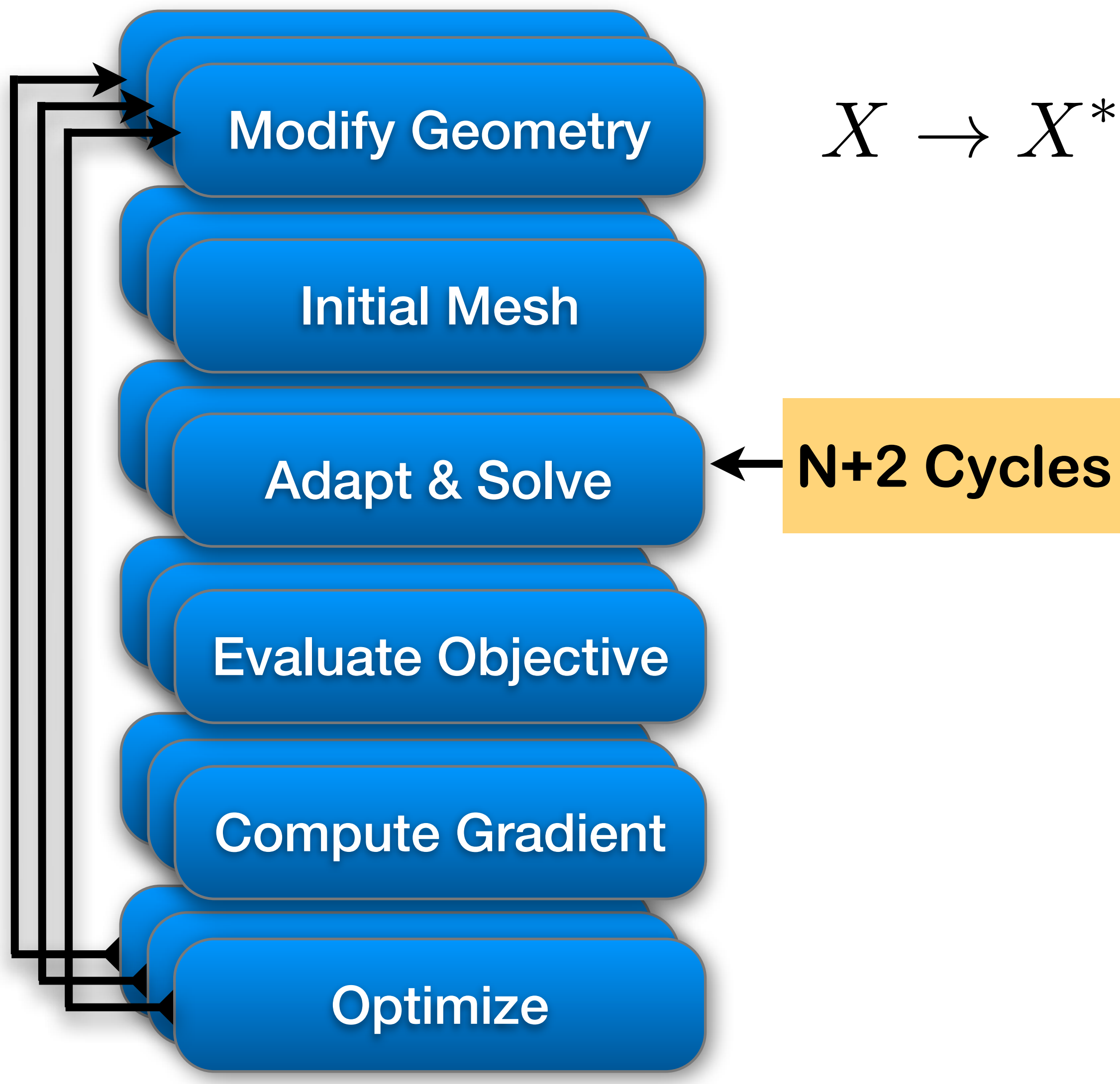

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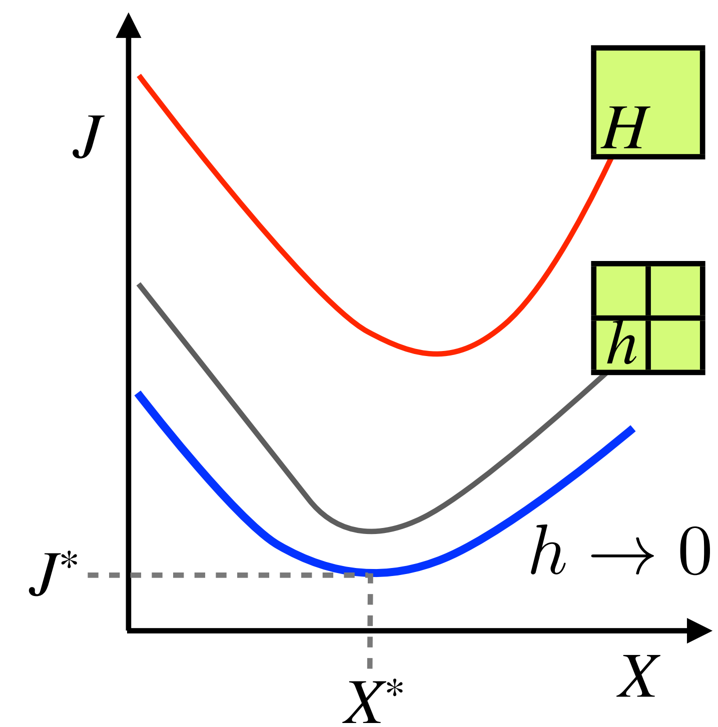
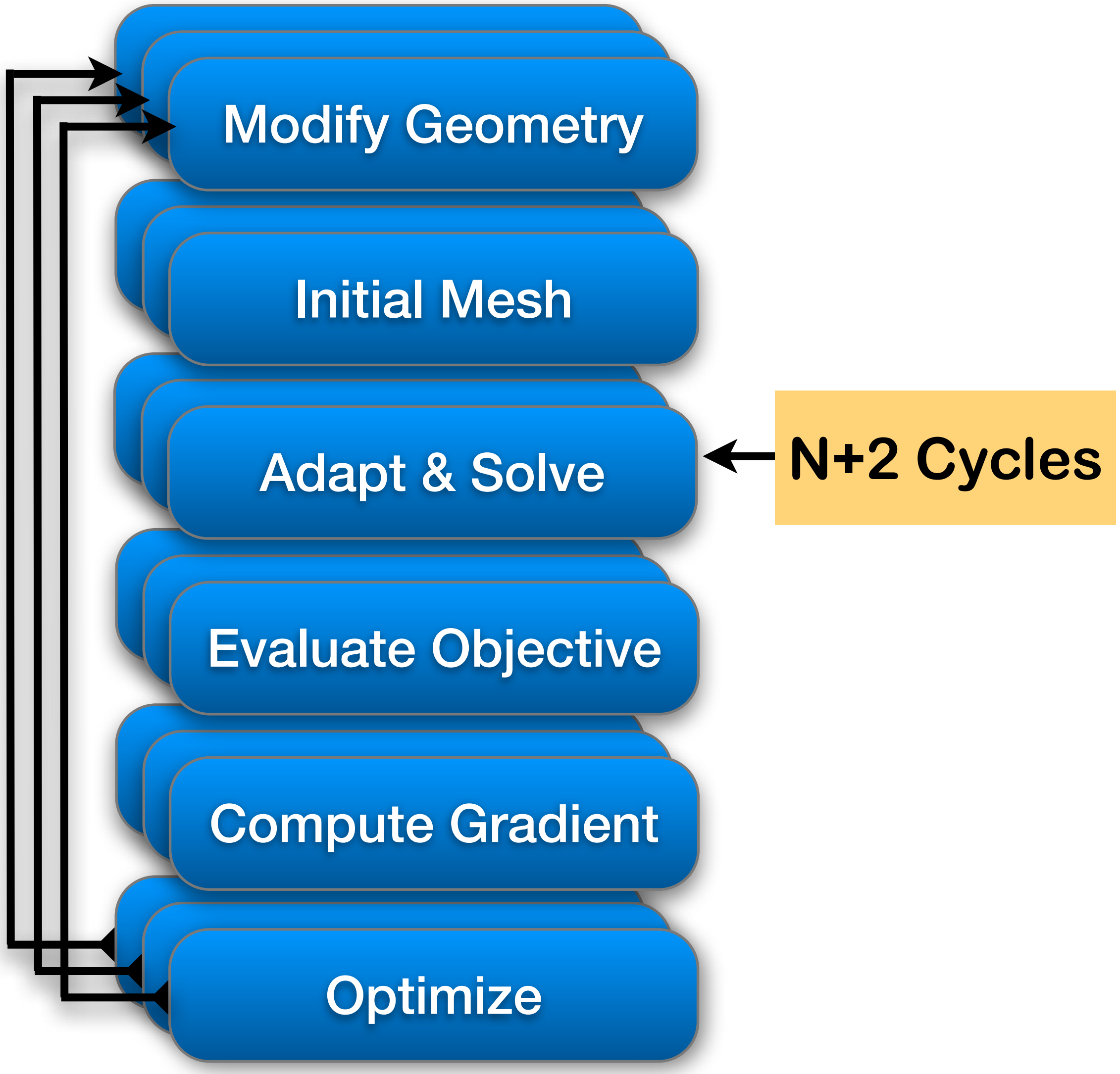


Progressive Optimization

- Increase mesh refinement in each optimization subproblem
  - Converge a sequence of improving discretizations

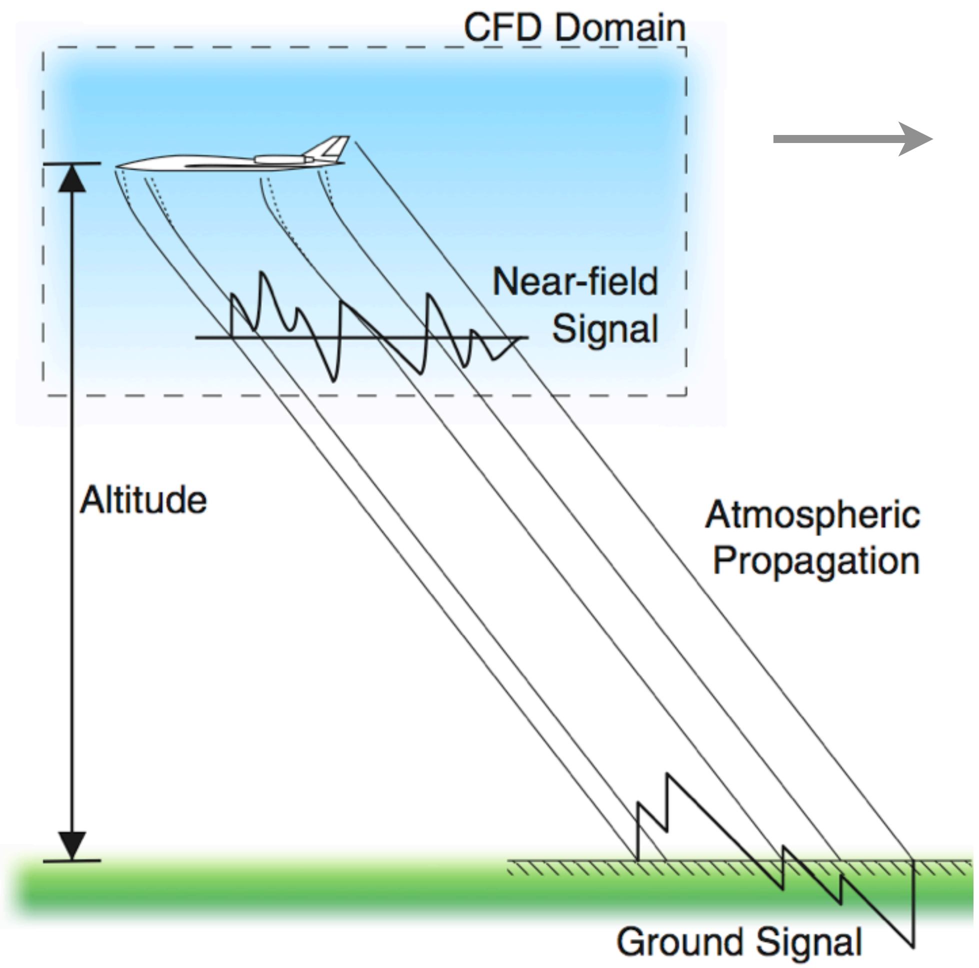
- Stopping Criterion
  1. Gradient or KKT norms, or stall
  2. Specified number of search directions
  3. Diminishing changes in objective function
  4. Ratio of design improvement to error: refine when $J_{i-1} - Ji < \mathcal{E}$

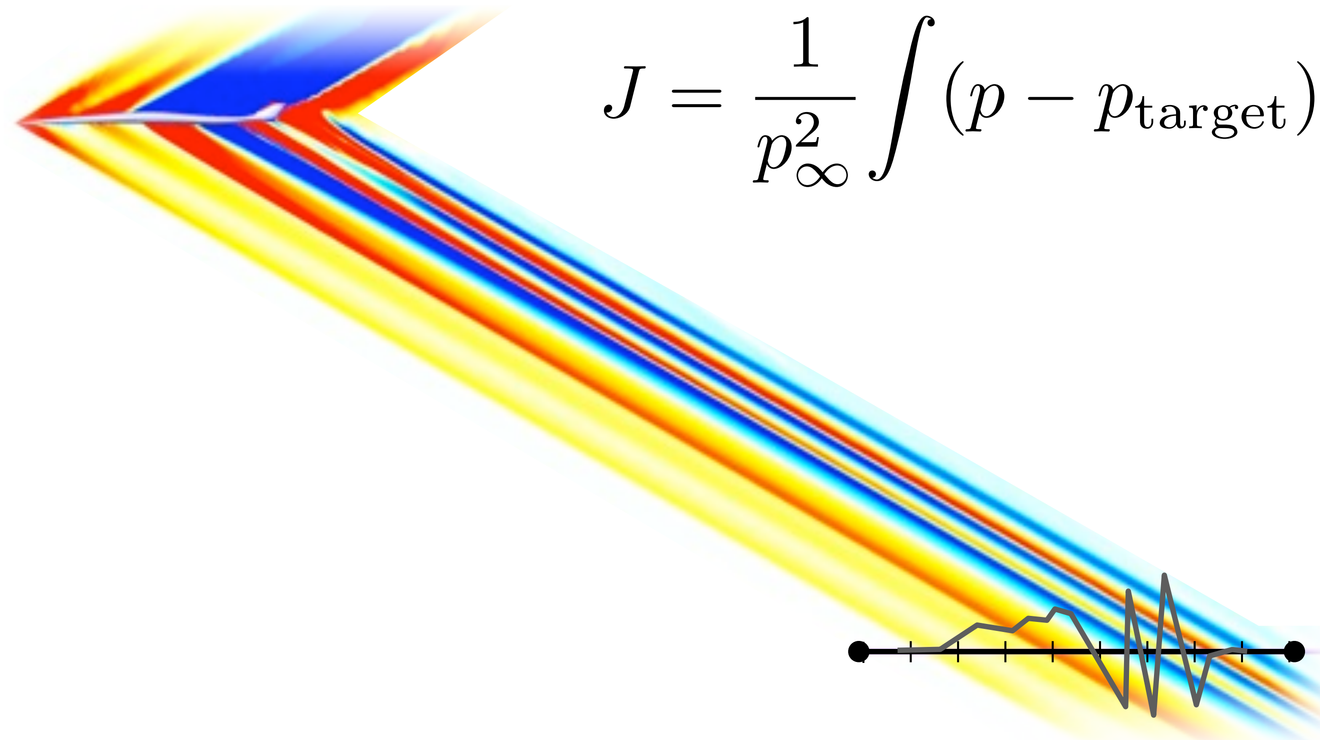


Results

Sonic-Boom Mitigation Inverse Design

Optimize aircraft shape by prescribing quieter near-field signals

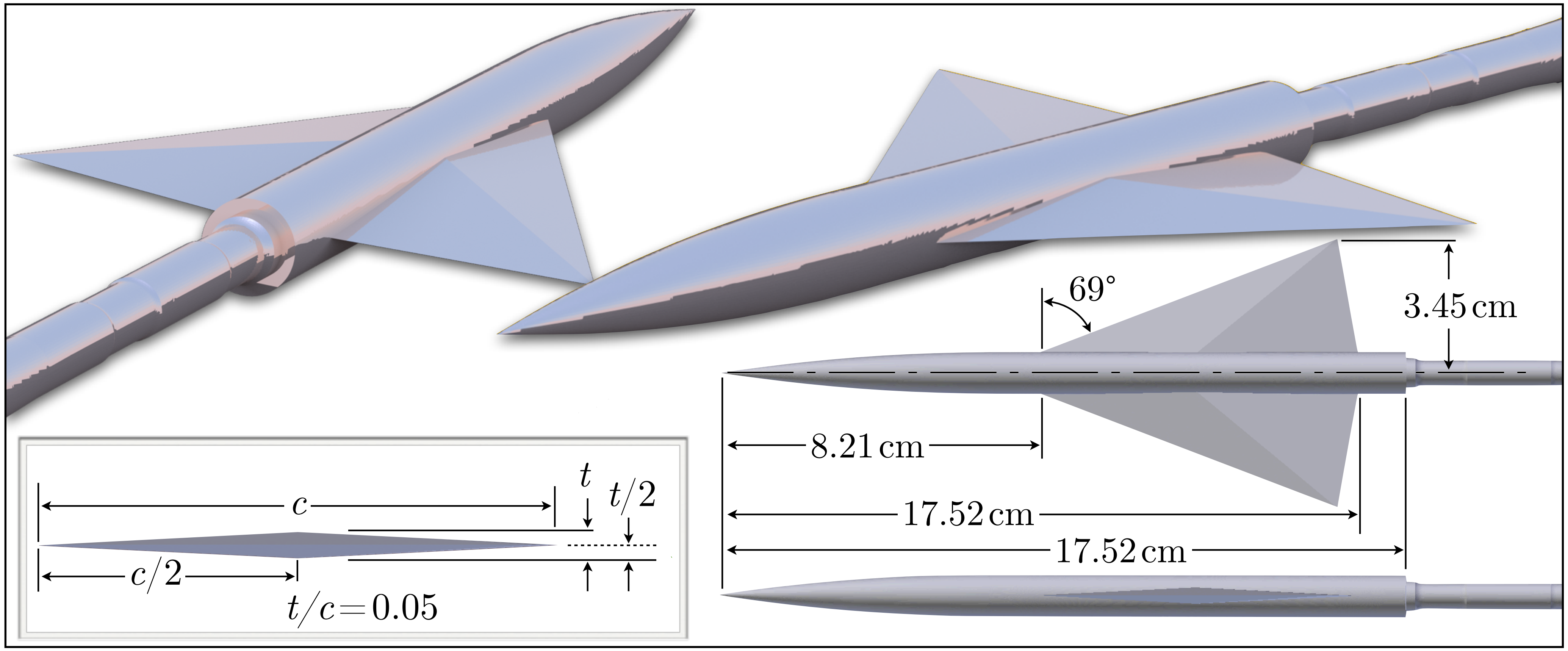




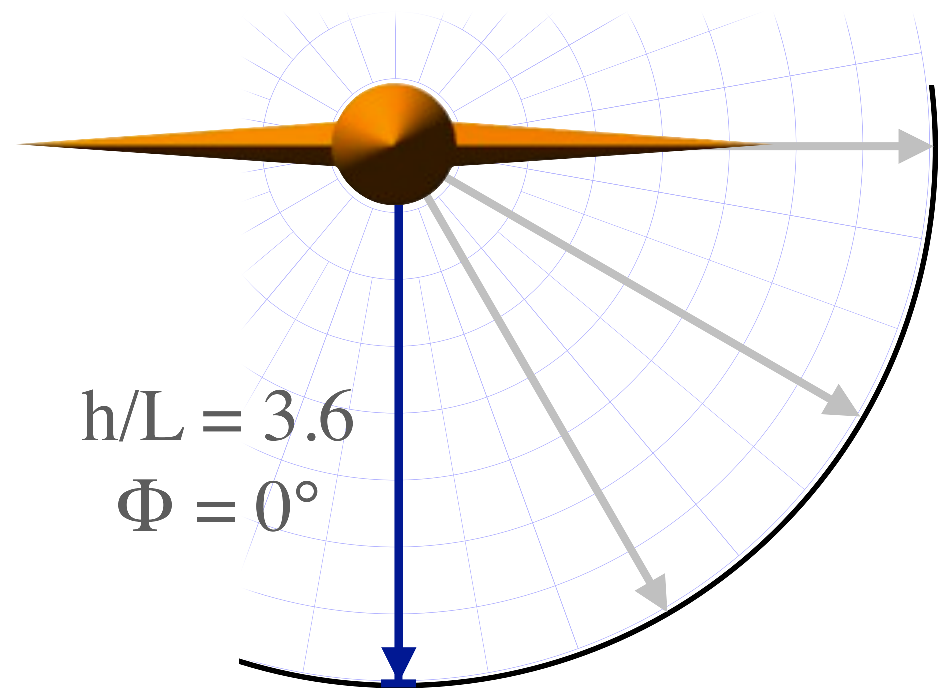
$$J = \frac{1}{p_\infty^2} \int (p - p_{\text{target}})^2 dS$$

1. Pressure-signature analysis
2. Shape optimization on a fixed mesh
3. Progressive optimization

Pressure Signature of Delta-Wing Body



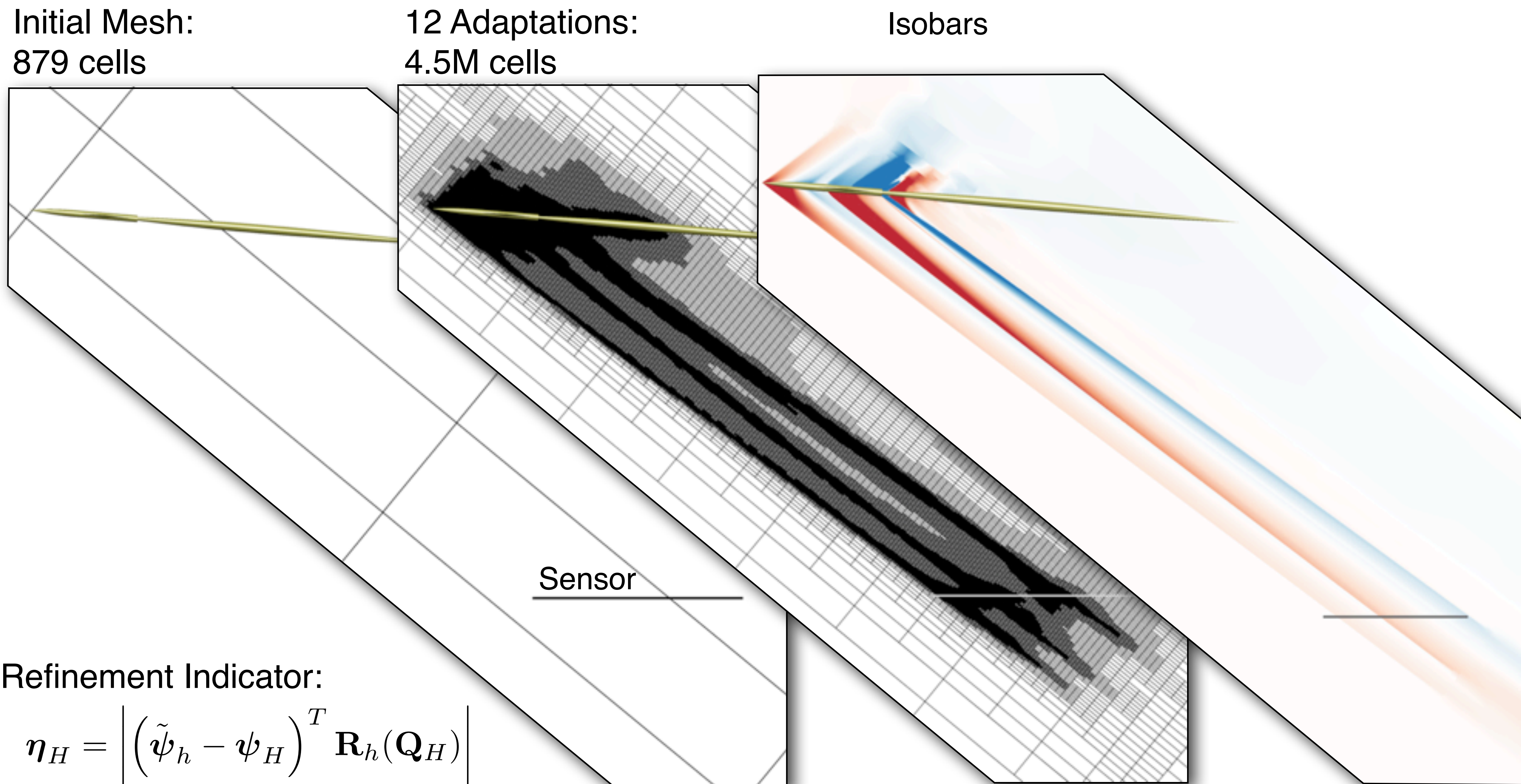
Determine pressure signature 3.6 body-lengths below the model



Freestream Conditions:

- $M_\infty = 1.68$
- $C_L = 0.15$

Mesh and Solution



Refinement Indicator:

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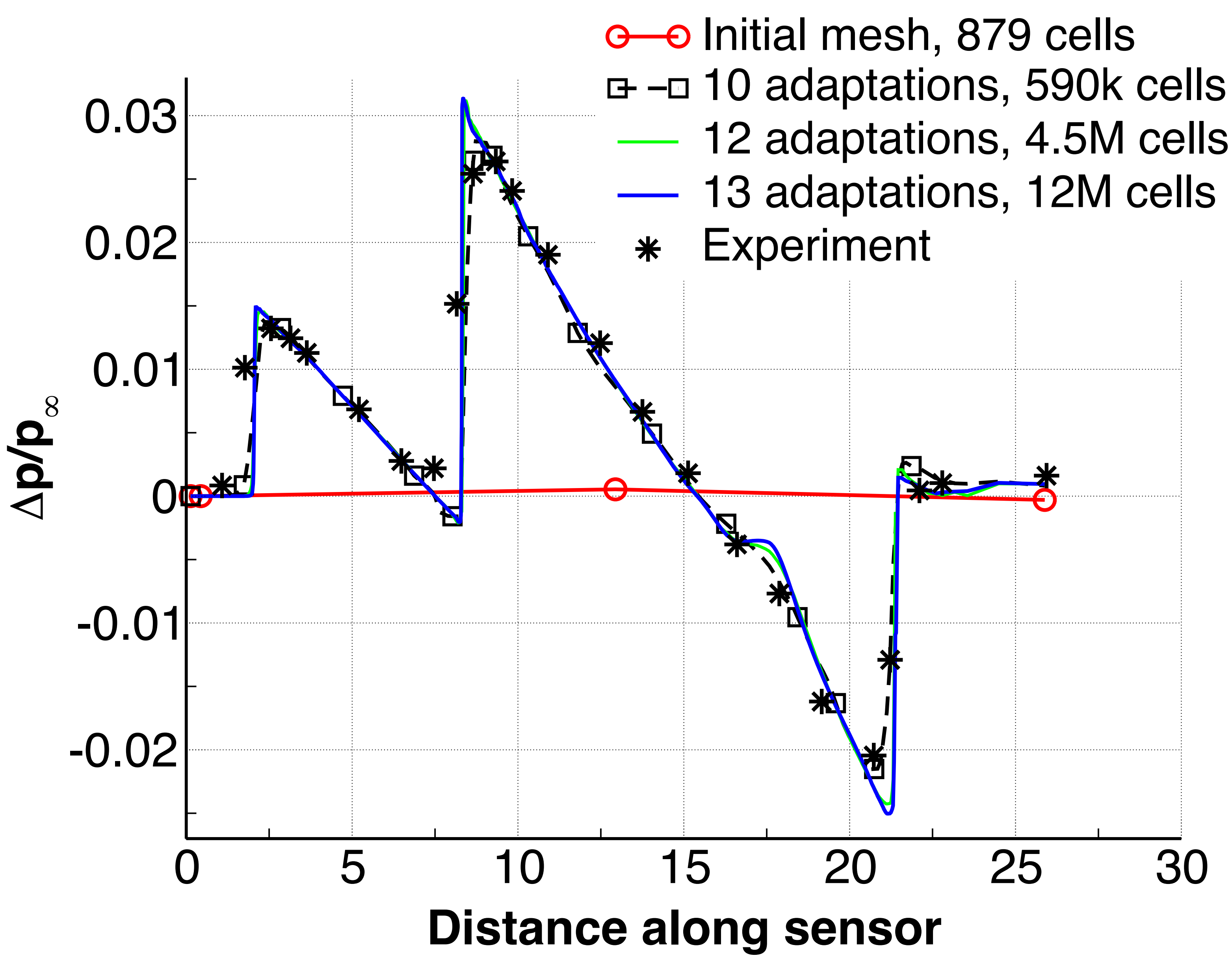
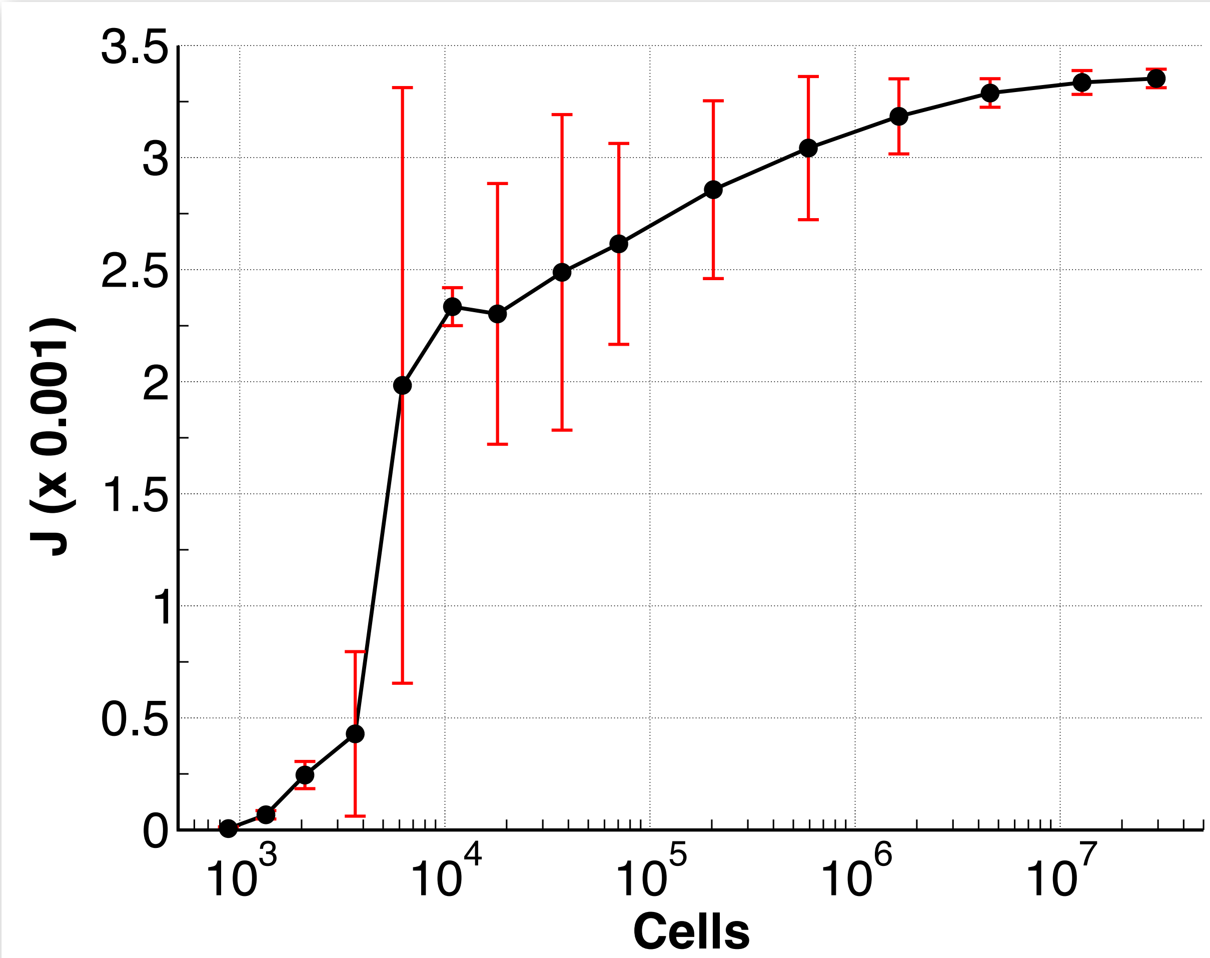
Near-field on symmetry plane

Pressure Signature



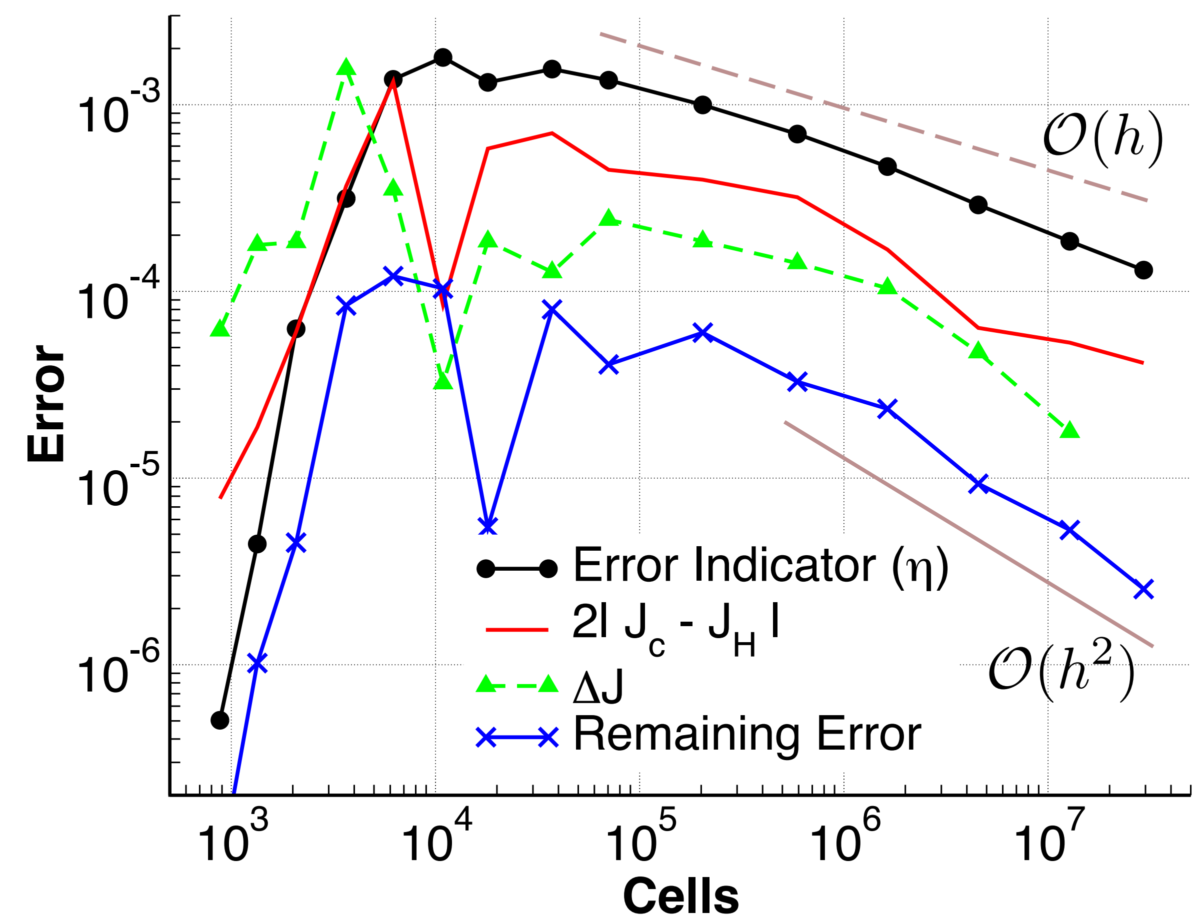
Error Convergence



$$J = \frac{1}{p_\infty^2} \int (p - p_\infty)^2 dS$$

- Error bars represent level of discretization error

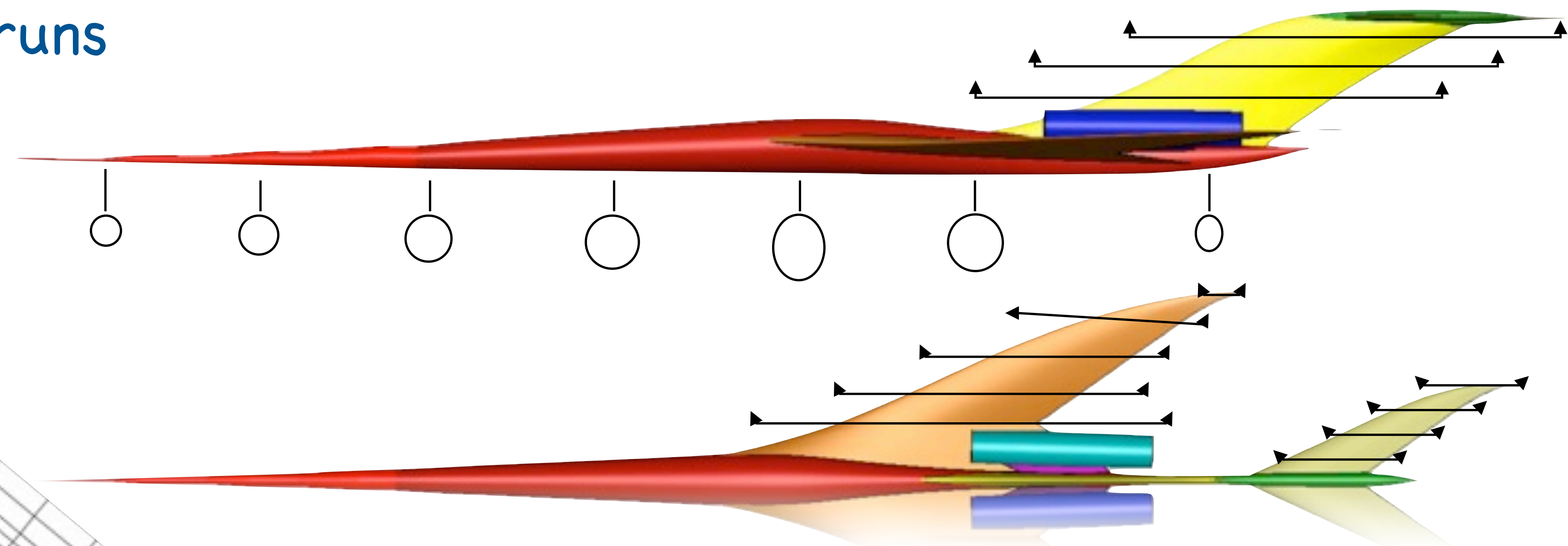
$$\mathcal{E} = 2\,|J_c - J_H|$$



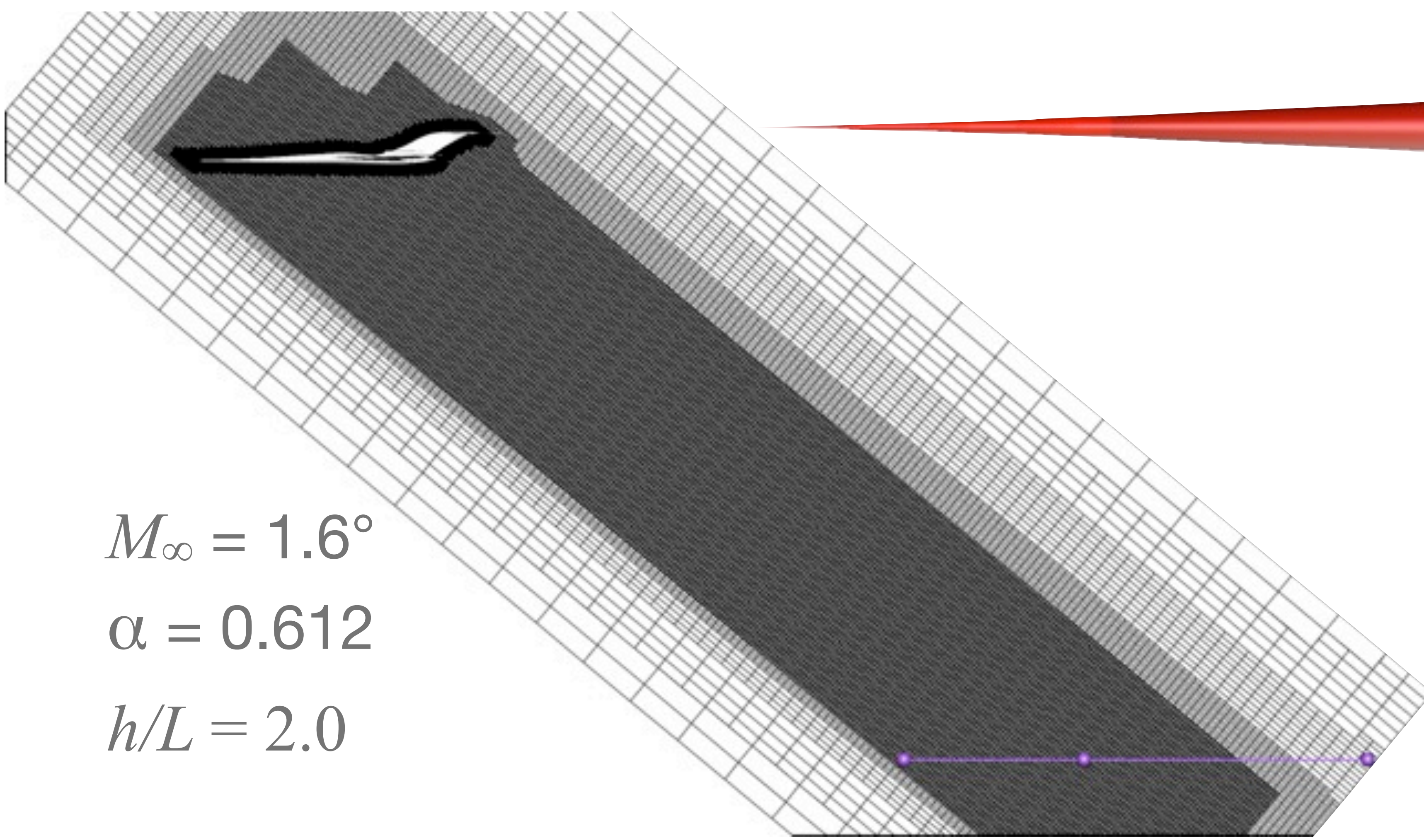
- Remaining error term is small and is $\mathcal{O}(h^2)$
- Error indicator is $\mathcal{O}(h)$ (due to localization)

Inverse Design on Fixed Meshes

Approach: use adaptation to guide construction of a fixed mesh for shape optimization runs



9.3 M Cells



$M_\infty = 1.6°$

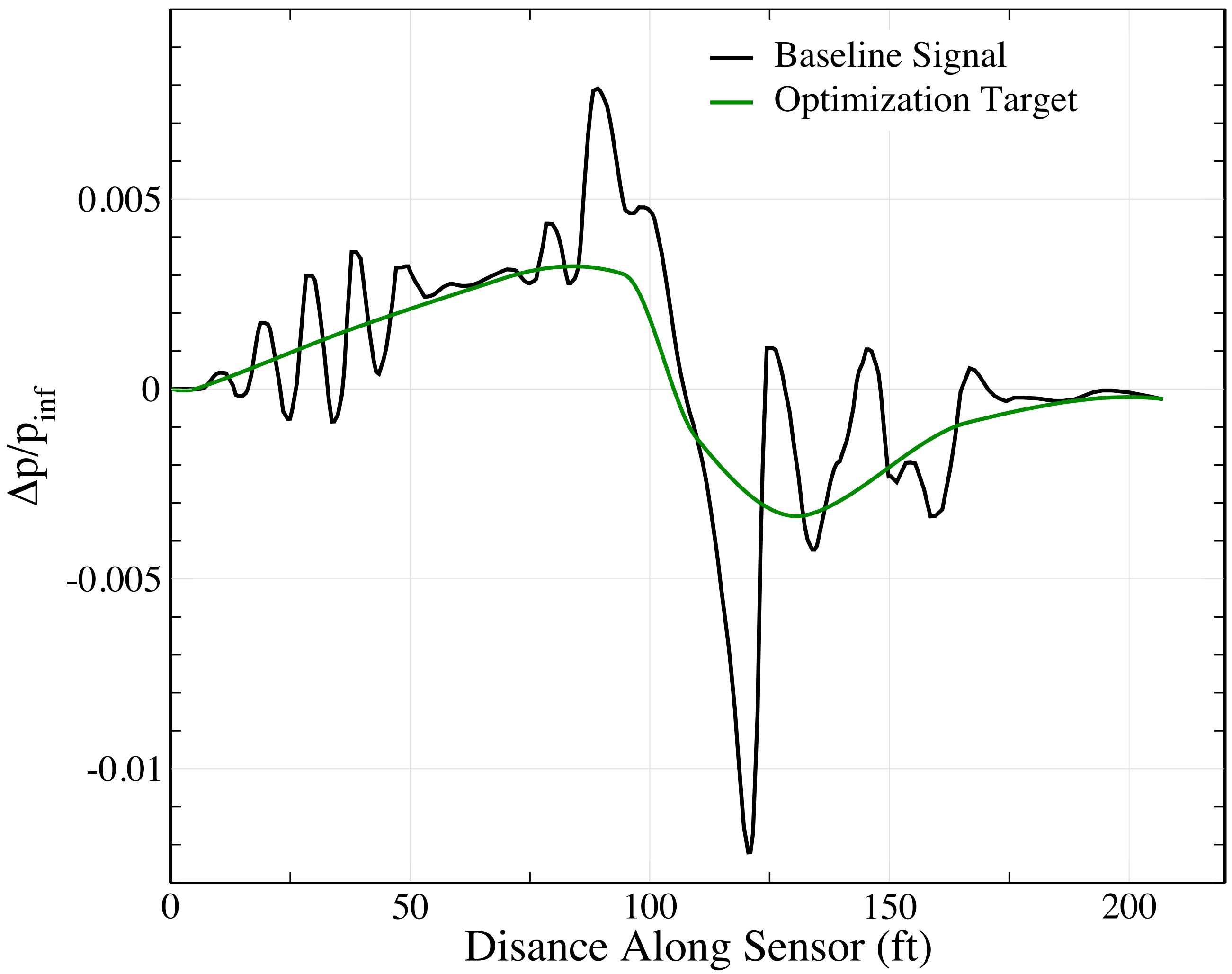
$\alpha = 0.612$

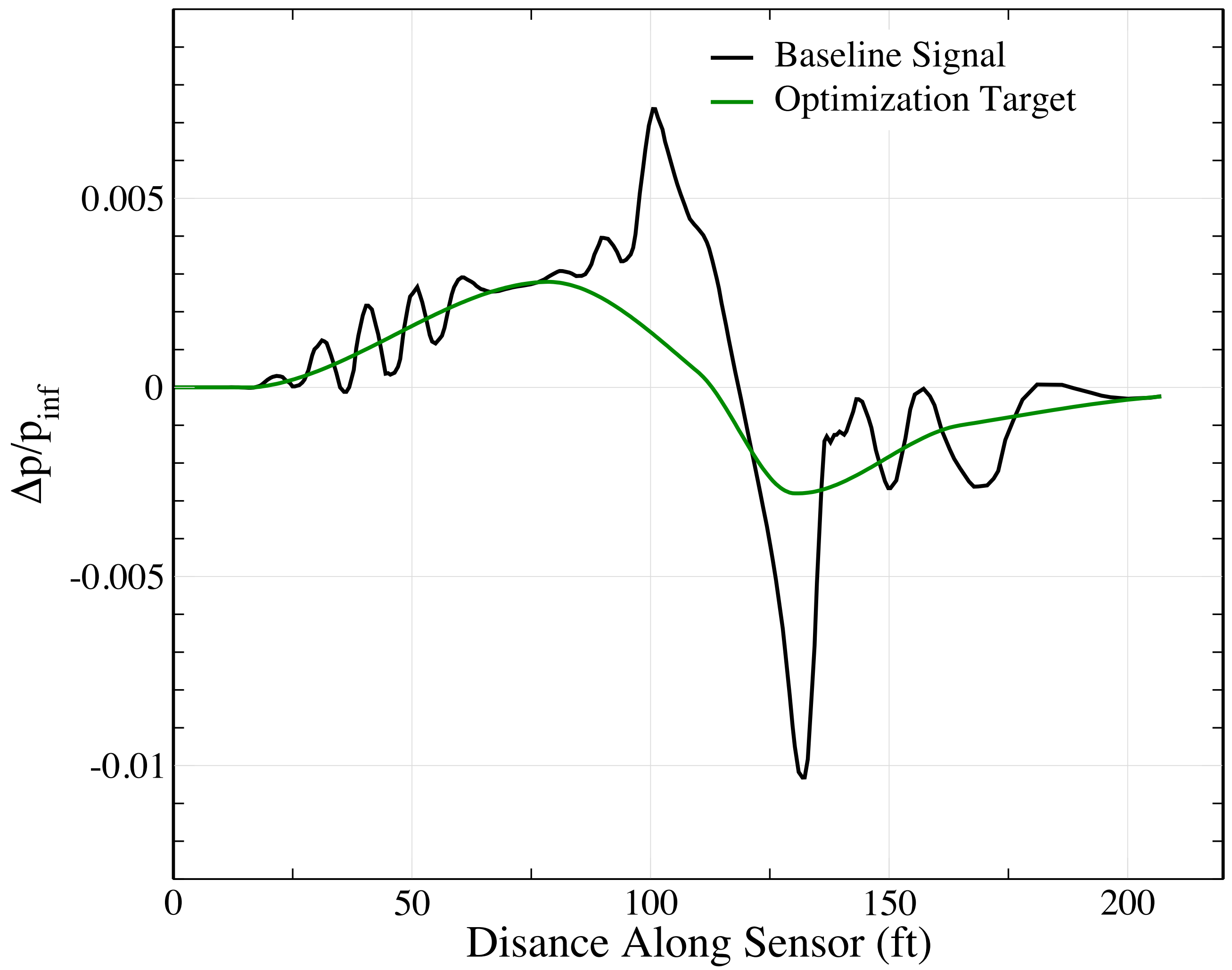
$h/L = 2.0$

Full aircraft configuration:
180 design variables

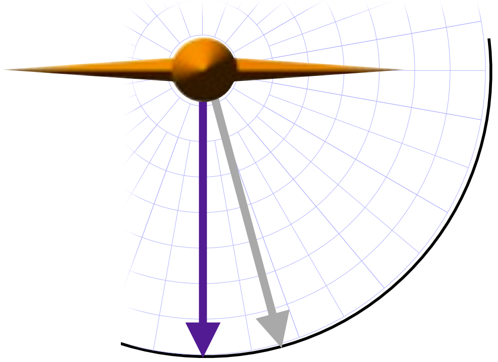
Optimization Targets

$$J = \frac{1}{p_\infty^2} \int (p - p_{\text{target}})^2 dS$$

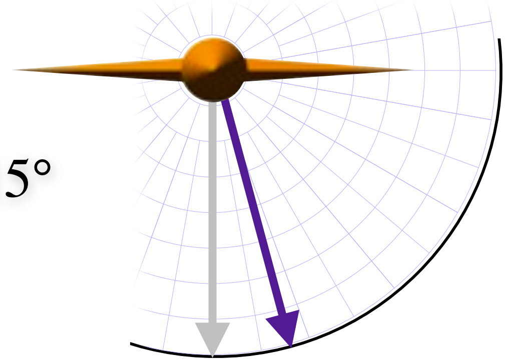




On-track, $\Phi = 0°$

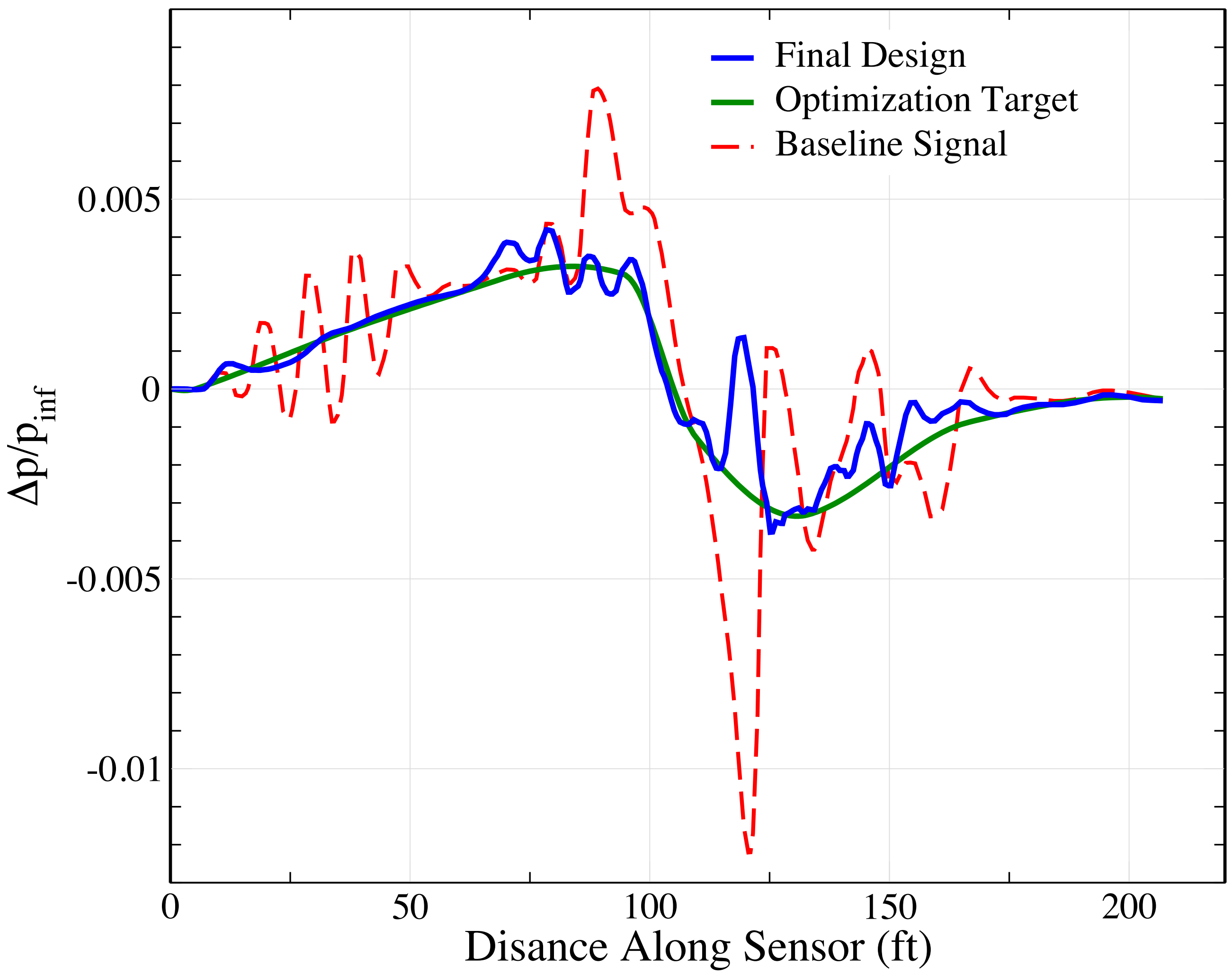


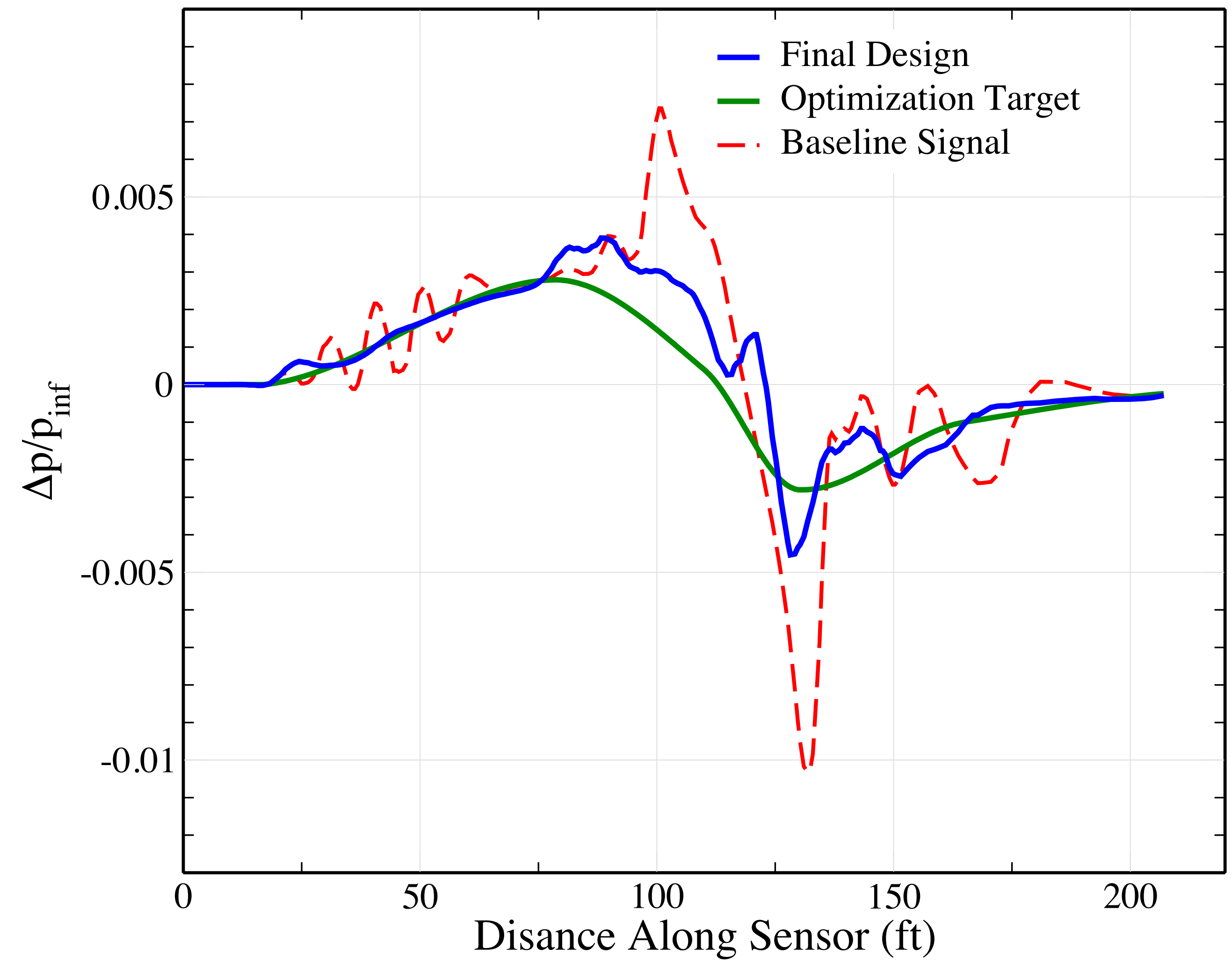
Off-track, $\Phi = 15°$

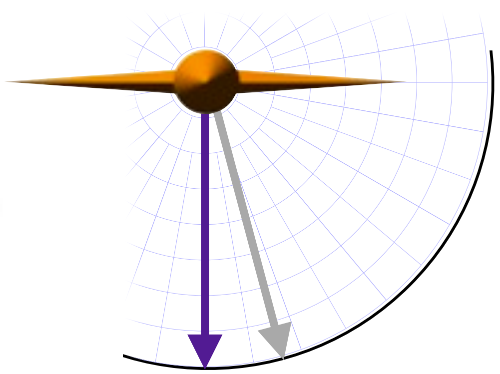


Optimization Results

- 50 design iterations (SNOPT)
- Ground noise 76.7 PLdB, 9.6 dB reduction in perceived loudness







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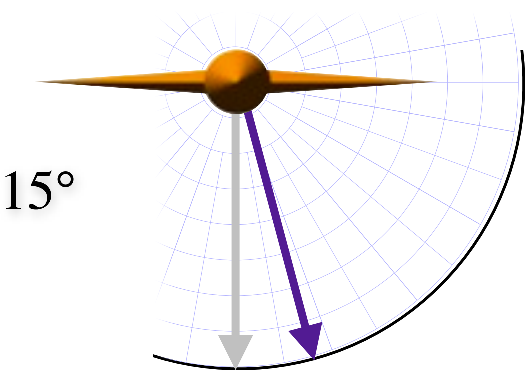
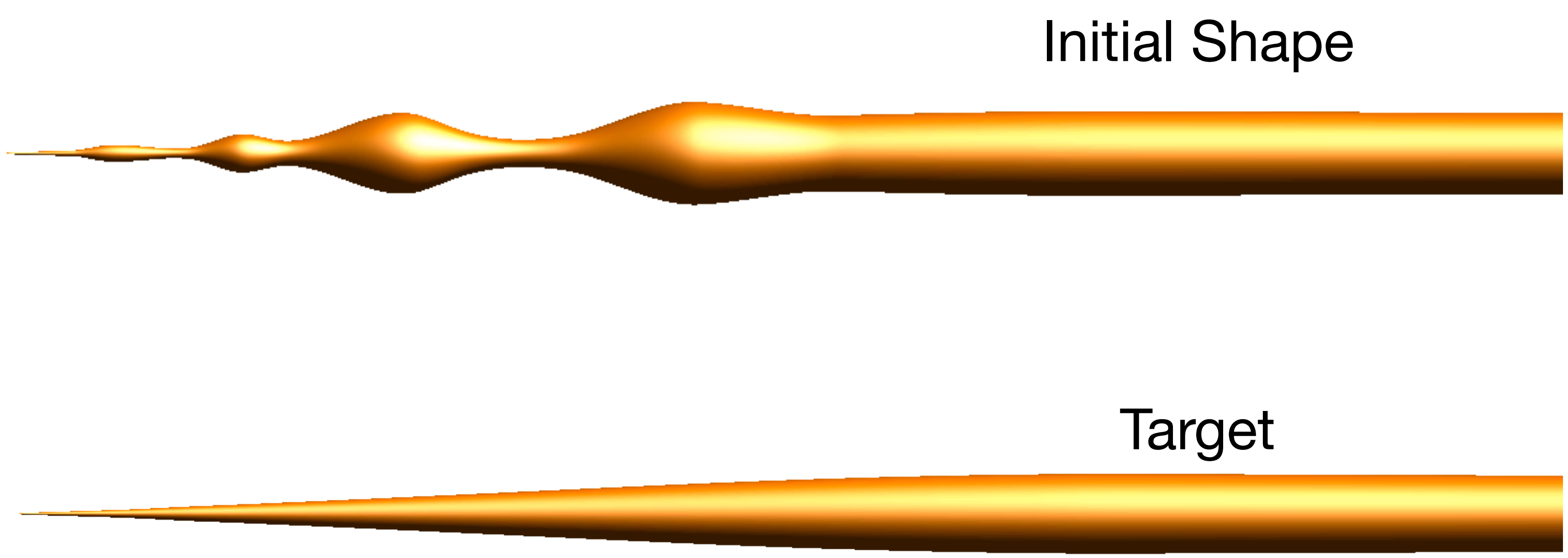


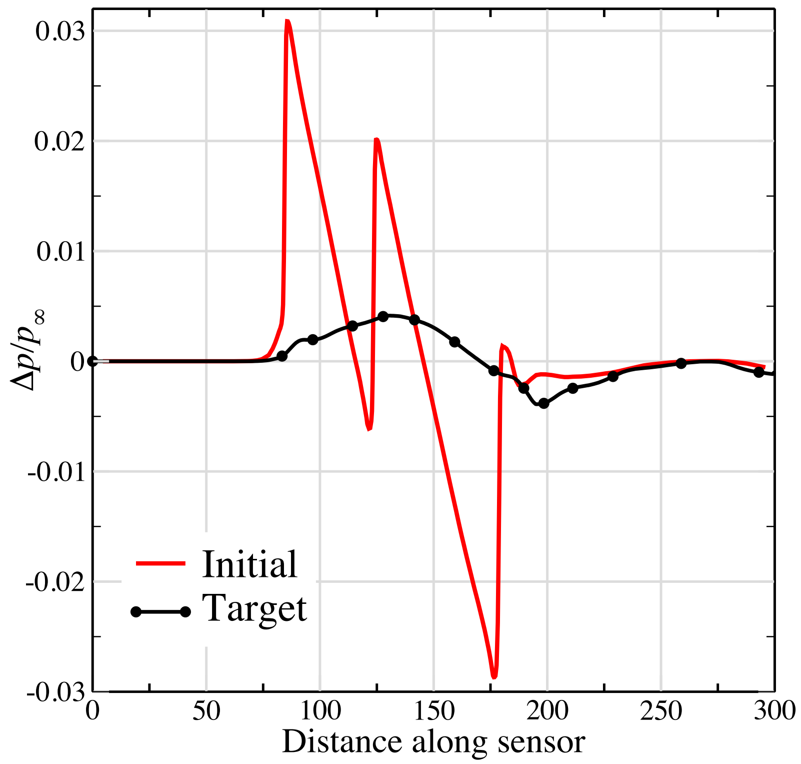
Off-track, $\Phi = 15°$

Optimization with Adaptation

Model Problem Setup

- Prescribe a target signature from a known shape
- 10 design variables that control body radius
- $M_\infty = 1.5$ and $\alpha = 0°$





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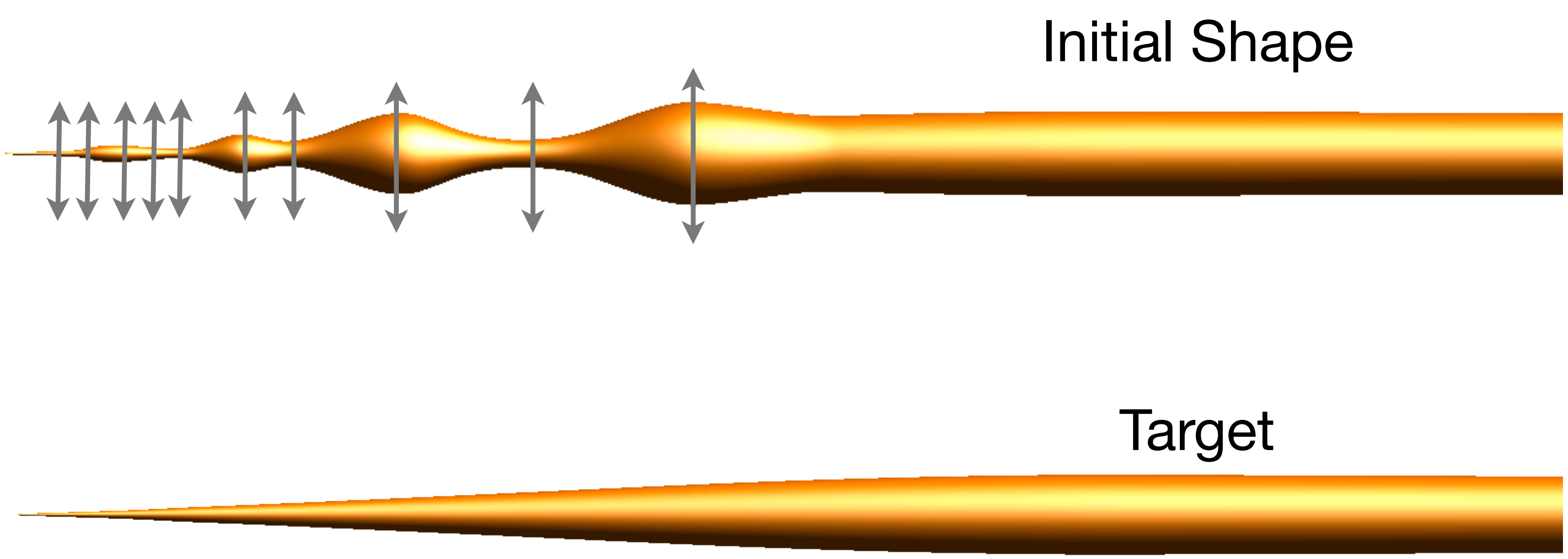
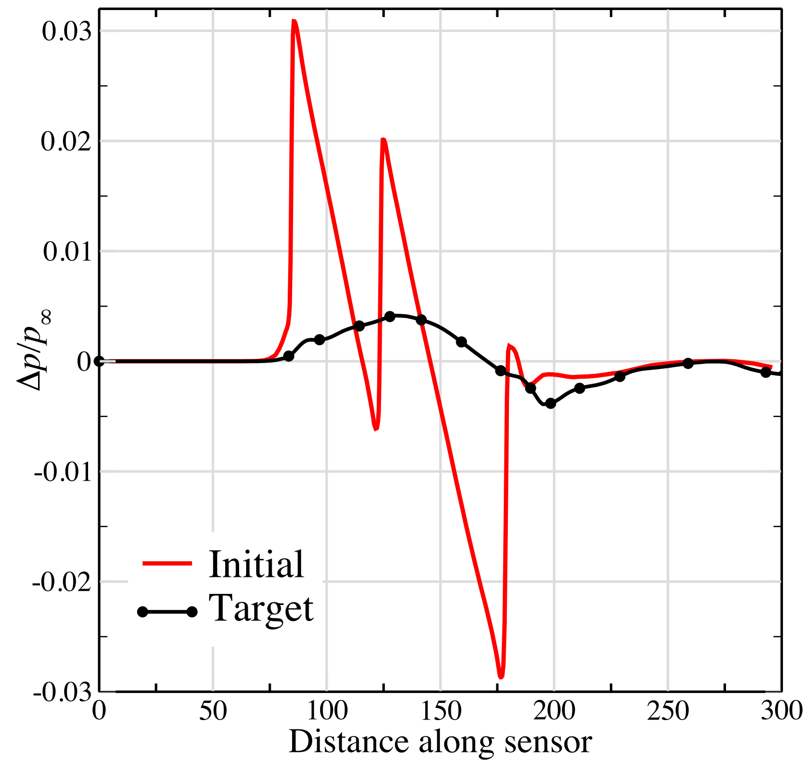
- $M_\infty = 1.5$ and $\alpha = 0°$

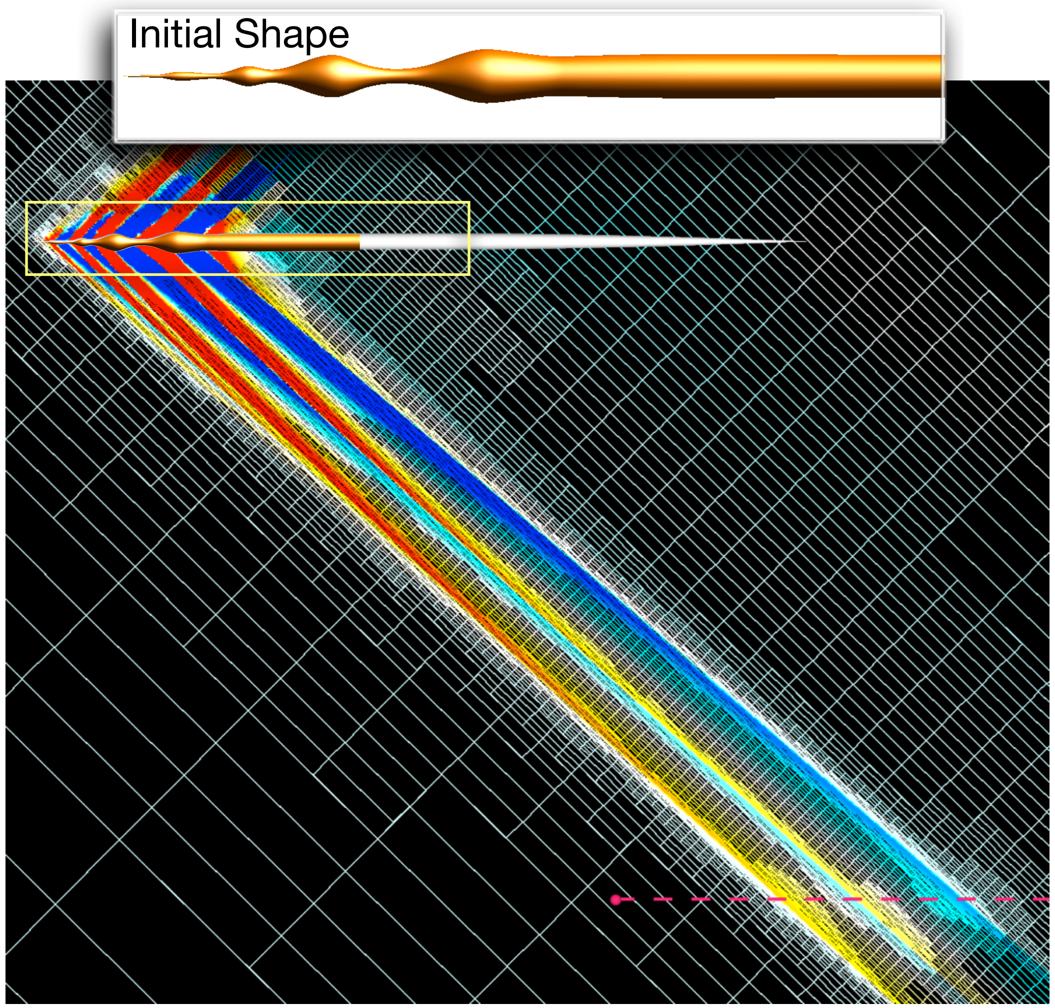


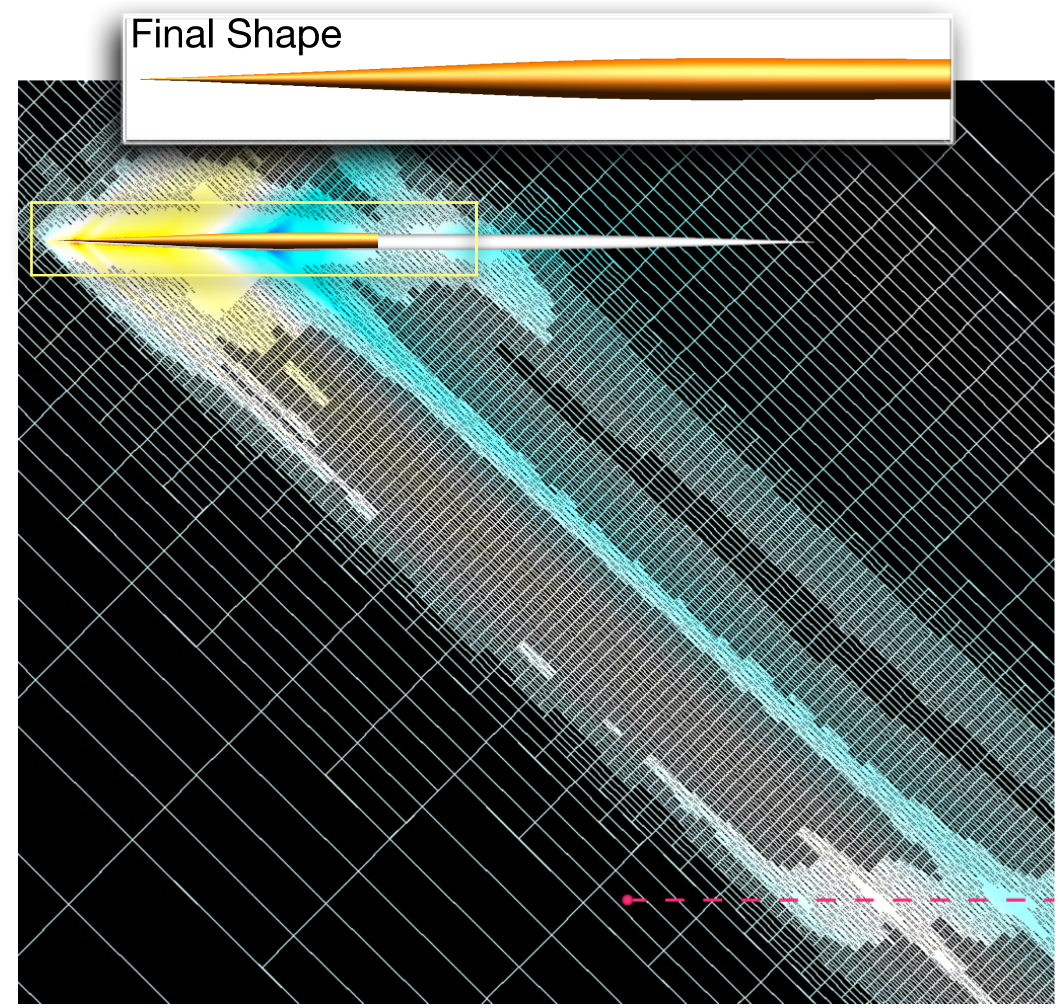


Optimization with Adaptation

Consider two cases

1. **Fixed-depth strategy:** 7 refinements in each design iteration
2. **Progressive optimization:** Increment from 4 to 7 refinements (allow designs to advance as far as possible on each level)

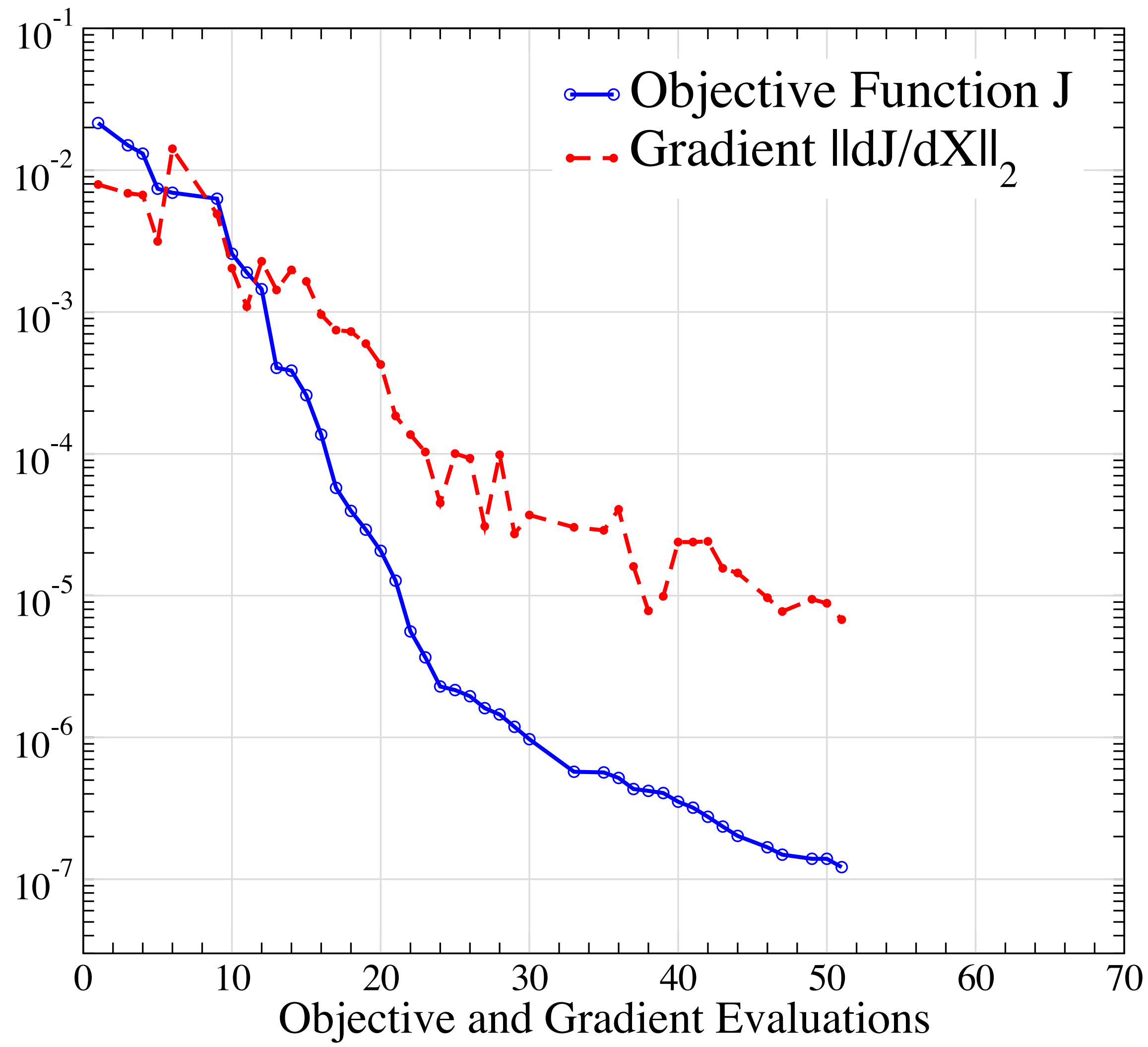




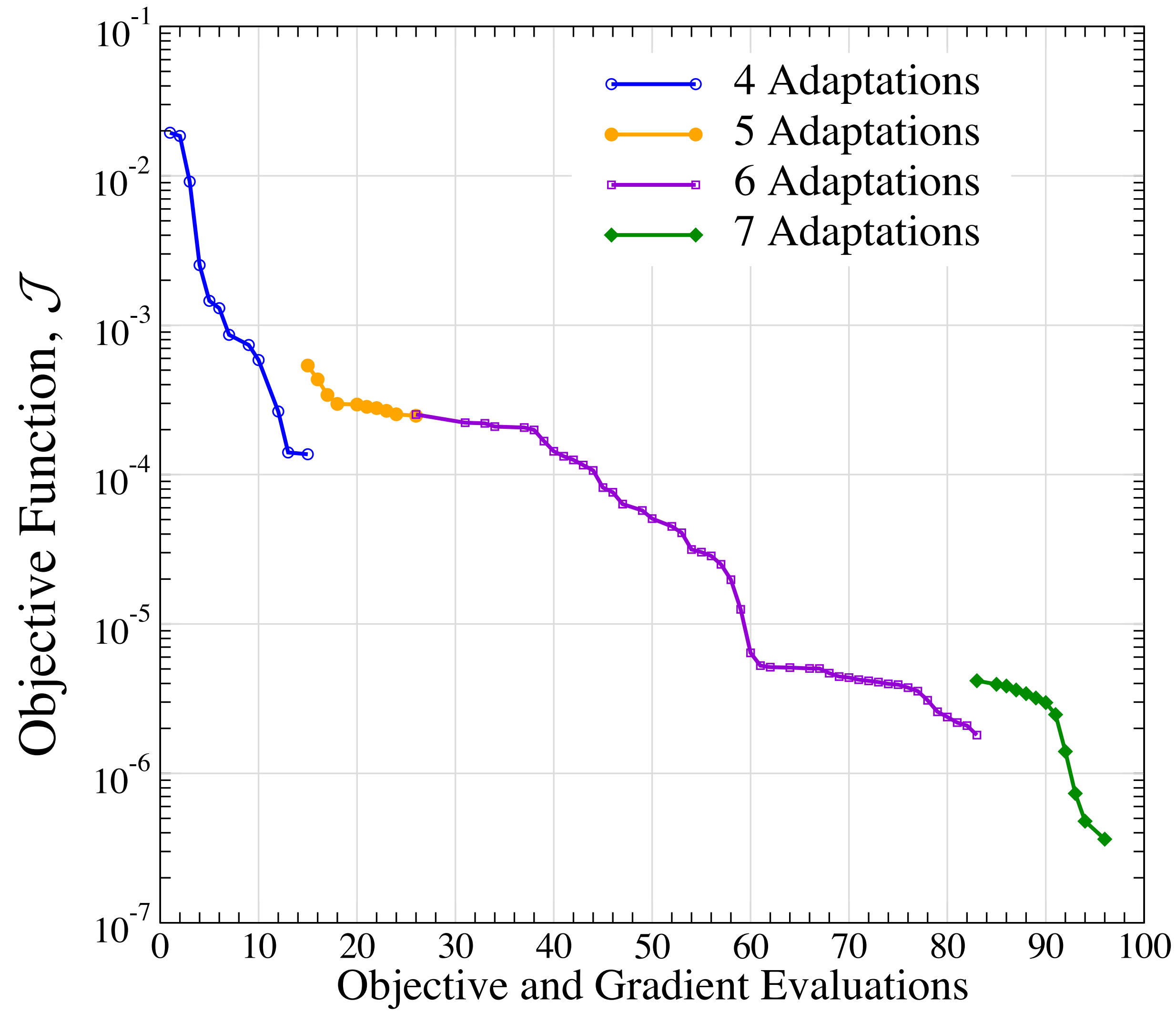
7 Adaptations, ~650k cells

Optimization with Adaptation

Fixed-Depth Strategy
7 Refinements



Progressive Optimization



Progressive optimization is about a factor of two faster than fixed-depth strategy

Summary and Outlook

- Progress toward a gradient-based optimization framework with capability to perform adaptive meshing in each design iteration
  - Promising approach to enhance accuracy, efficiency and automation of simulation-based design
- Future work
  - Use of error estimates to limit oversolving
  - Transfer of Hessian matrix as the design moves from mesh to mesh
  - Dynamic error control and mesh re-use

Questions

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Cart3D website and publications:

http://people.nas.nasa.gov/aftosmis/cart3d/